Micro Comp Second Round June 27, 2016

Part A: Short Questions

1. (Jones) Ted's expenditure function is $e(\mathbf{p}, u)$. His demand function for ice cream is $x_j(\mathbf{p}, y)$, where $\mathbf{p}$ is vector of prices and $y > 0$ is his income. Show that ice cream is a normal good for Ted if and only if $\partial^2 e/\partial p_j \partial u > 0$.

2. (Jones) Let u and v be two utility-of-wealth functions, with $v(w) = f\big(u(w)\big)$, where f is strictly concave. Is the following statement true or false? "The coefficient of absolute risk aversion for v is greater than for u."

3. (Pape) Suppose that two players play a repeated Prisoner's Dilemma. Suppose after every play of the PD, there is a half chance that the game is over, and the players do not continue, and there is a half chance that play continues and they play the PD again. Is it possible to achieve cooperation in this context? Why or why not?

4. (Pape) Consider an economy of multiple agents with quasilinear preferences over a consumption good, *x*, and a public good, *G.* Show that the socially optimal level of G exceeds the privately provided level. Explain.

Part B: Medium-Length Questions

5. (Jones) Prove:
   a. When the production function is homothetic, the cost function is multiplicatively separable in input prices and output and can be written $c(\mathbf{w}, y) = h(y)c(\mathbf{w}, 1)$, where $h(y)$ is strictly increasing and $c(\mathbf{w}, 1)$ is the unit cost function.
   b. For homothetic production functions, the output at which average cost is a minimum is independent of factor prices.

6. (Jones) Consider a two-period model with Lisa's utility given by $u(x_1, x_2)$ where x_1 represents her consumption during the first period and x_2 is her second period's consumption. Lisa is endowed with $(\bar{x}_1, \bar{x}_2)$ which she could consume in each period, but she could also trade present consumption for future consumption and vice versa. Thus, her budget constrain is

$$p_1x_1 + p_2x_2 = p_1\bar{x}_1 + p_2\bar{x}_2,$$

where p_1 and p_2 are the first and second period prices respectively.

a. Derive the Slutsky equation in this model.
b. Assume that Lisa's optimal choice is such that $x_1 < \bar{x}_1$. If p_1 goes down, will Lisa be better off or worse off? What if p_2 goes down?
c. What is the rate of return on the consumption good?

7. (Pape) Consider an Edgeworth box of a simple exchange economy between two agents, A and B, over two goods, X and Y, and there is a fixed endowment vector ω, in which each agent has a positive amount of each good. Suppose that A has a kind of market power in which she can choose prices, after which A and B trade as much or as little as they wish at those prices. Either show how A may choose prices that are not consistent with Walrasian equilibrium or show that A will choose prices that are consistent with Walrasian equilibrium.

8. (Pape) In a game show, Rowland and Colette are the two players, and they have earned *z* dollars in cash and prizes. Now they are in the final round of the game. In this game, they each simultaneously choose **Split** or **Steal.** If they both choose Split, they each take home half of the winnings, .5*z*. If one chooses Split and the other Steal, then the one who chose Steal takes home .9*z* and the other only takes home .1*z.* If they both choose Steal, then they take home a fraction *x* of what they would have taken home if they both choose Split; i.e. .5*x z. x* is between zero and one and known to both players. Find all NE of this game as a function of *x*.

Part C: Long Questions

9. (Jones) Consider the insurance signaling game with 2 types of consumers, low- and high-risk ones. Let α be the probability that a consumer is low-risk. Show that when α is sufficiently large, there exist pooling equilibria that make both consumer types better off than they would be in every separating equilibrium. What is the intuition behind this result?

10. (Jones) Consider a first-price, seal bid auction in which the seller sets a reserve price. Specifically, assume there are only 2 bidders whose values are drawn independently from a common distribution on [0, 1]. The seller announces a reserve price $c \in [0,1]$. If the

highest bid is at least c, the object will be sold to the highest bidder at a price equal to his bid. If the highest bid is less than c the object will not be sold.

a. Show that if a bidder's value is less than c, bidding 0 is optimal.
b. Part (a) implies that a bidder's bid can be positive only if his value is greater than or equal to c. Find the unique symmetric equilibrium bidding function $\hat{b}(v): [c, 1] \rightarrow \mathbb{R}_+$. (**Hint**: We can set $\hat{b}(c) = c$ because if a bidder's value is c then bidding c is optimal.)
c. When $c > 0$, does a bidder with value $\in [c, 1]$ bid higher or lower than in an ordinary first-price auction?
d. Suppose the bidders' values are drawn from a uniform distribution on [0, 1]. What is the equilibrium bidding function? What is the seller's expected revenue? What value of c maximizes expected revenue?

11. (Pape) Two economists, A and B, have decided to work on a paper together. There are three important variables in the game: (1) first author *F* either A or B, determined below. The first author is considered a greater "owner" of the project and also receives more fame from the project. (2) Effort level $e_i \geq 0$ put in by each player *i*, which describes hard the player works on this project, and (3) significance $s \geq 0$ that the paper eventually achieves.

    The game proceeds in two rounds of play. In the first round, agents simultaneously choose action a_i = "ME," or "NO PREFERENCE." If player *i* chooses "ME," he is insisting on being first author. To choose "NO PREFERENCE" indicates agreement to the other player being first author. If both players choose "ME" then they cannot agree on author order and the game immediately ends with a payoff of 0 for each player.

    In the second round, if reached, agents simultaneously choose a level of effort $e_i \geq 0$.

    Then outcomes are calculated. First, *s* is determined in the following way: $s = e_A \bullet e_B$. Then, author order is calculated: if player *i* chooses *ME* and the player *j* chooses NO PREFERENCE, then *i* is chosen (i.e. F=*i*). If both players selected NO PREFERENCE, then the first author is chosen randomly. Finally, payoffs are calculated for first author *F* and then the second author $j \neq F$.

    $u_i(s, e_F) = 2\ln(s) - ½ e_F^2$
    $u_j(s, e_j) = \ln(s) - ½ e_j^2$

    Find all pure-strategy SPNE of this game.

12. (Pape) Consider the following economy: There is a single consumption good, *x*, which is produced by the single factor of production, labor *L*. There are 50 identical consumers, each endowed with $\gamma>0$ units of labor and zero units of x. Consumers have the utility function $u(x,t) = xt$, where t is units of leisure (i.e. "labor" that is not sold to firms.) There are two kinds of firms: firms of type A have the following production function: $f_A(L_A) = L_A^{1/2}$. Firms of type B have the following production function: $f_B(L_B) = .1\ L_B$. There are 50 of each type of firm, and each consumer owns exactly one type A firm and one type B firm. Question: In Walrasian equilibrium, will all firms participate in production of *x*? Possible answers: Yes, No, or Sometimes. Completely explain the answer you choose, and if your answer is Sometimes, make sure to precisely describe the circumstances when all firms do, versus all firms do not, participate.