Welfare Cost of Inflation in a General Equilibrium Model with Currency and Interest Bearing Deposits

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Abstract: The user cost of non-interest bearing currency is the nominal interest rate. Increases in inflation lead to decreased real currency held in steady state, which imposes a welfare cost. Lucas (2000) estimates this welfare cost in a model with a single non-interest bearing monetary asset. Lucas suggests applying monetary aggregation/index number theory to generalize the model to contain interest-bearing deposits. We solve a general equilibrium model with three agents: a household, a goods producing firm, and a financial firm. We assume that currency and deposits provide utility to the household and form a weakly separable group in the utility function. We use monetary aggregation and index number methods to calibrate the model and estimate the welfare cost of inflation. We compare these estimates to welfare cost estimates from a model in which all money is non-interest bearing currency. We find that the welfare cost of inflation is substantially lower in the models with interest-bearing deposits than in models with only non-interest bearing currency. We also find that the welfare cost of inflation is positively related to the own price elasticity of aggregate money demand.

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The user cost of non-interest bearing monetary assets is the nominal interest rate. An increase in the rate of inflation will cause the nominal rate of interest to increase and therefore makes non-interest bearing assets less attractive relative to interest-bearing assets, consumer goods, and leisure. The attempt to substitute away from non-interest bearing monetary assets in response to inflation will cause the price level to rise and the real stocks of the assets to fall. This is the basic idea underlying a long line of research on the welfare cost of inflation, recently surveyed and updated in Lucas (2000). Lucas (2000) solves for the steady state in a general equilibrium money-in-the-utility function model. In his model, steady state consumption is invariant to the rate of inflation and the real stock of non-interest bearing money in steady state (and hence the steady state instantaneous utility) is inversely related to the rate of inflation. The welfare cost of inflation can be estimated as the increase in steady state consumption necessary to offset the loss of utility caused by inflation, or by the closely related concept of equivalent variations.\textsuperscript{1} Other recent studies that address the welfare cost of inflation in different types of models are Gillman (1993), Dotsey and Ireland (1996), and Simonsen and Cysne (2001).

Lucas (2000) focuses on the M1 level of monetary aggregation and assumes that all components of M1 are non-interest bearing. He suggests that interest-bearing assets could be introduced into the model by applying monetary aggregation theory to measure the aggregate quantity of money as a superlative quantity index, following Barnett (1978, 1980, 1987). The user cost of interest-bearing monetary assets is not the nominal interest rate, but rather the interest rate differential between the rate on an alternative non-monetary asset and the own rate on the interest-bearing asset. The percentage change in a superlative Divisia monetary quantity index is the sum of the expenditure share weighted percentage changes in the component quantities of the index. M1 has contained explicit interest-bearing components since the mid-1970s, and almost all of the economists that have constructed superlative quantity index numbers at the M1 level have assumed that non-interest bearing demand deposits earn implicit interest.\textsuperscript{2} The additional assets in broader monetary aggregates such as M2 and M3 are all interest bearing. Modifying the standard money-in-the-utility function model to incorporate interest-bearing deposits leads to several complications. First, the own price elasticity of the “money demand function” could be quite different for superlative monetary indexes than for traditional aggregates such as M1 and M2. This is quite significant to estimates of the welfare cost of inflation, which are closely approximated by the area under the inverse money demand function.
Second, the model needs to describe both the supply and demand for interest-bearing assets in general equilibrium and to connect the steady state inflation rate to the steady state user cost of deposits.

We introduce monetary aggregation theory into the basic money-in-the-utility function model of Lucas (2000). In our model, households supply labor and derive utility from consumer goods, currency, and a good we call financial services. A government creates non-interest bearing currency and the two types of goods are produced by profit maximizing firms. The supply of financial services is based on the model of King and Plosser (1984). We establish the equivalence between financial services and interest-bearing deposits, and calibrate our model under the assumption that currency and interest-bearing deposits are weakly separable from consumer goods in the household’s utility function. We use superlative monetary quantity indexes in our calibration of the model.

We find that the estimated welfare cost of inflation is much lower than would have been found if we had assumed that all monetary assets are non-interest bearing. The economic logic of this result is straightforward. The expenditure share on non-interest bearing currency is very low relative to the expenditure share on interest-bearing deposits (particularly for broad levels of aggregation) and the steady state user cost of interest-bearing deposits is invariant to the steady state rate of inflation. Thus, an increase in the rate of inflation will have a smaller effect on the aggregate price of money than it would have had if all money were non-interest bearing and consequently there will be smaller welfare losses.

In Section I, we describe the model; in Section II, we calibrate the model and compute the welfare cost of inflation; in Section III, we outline possible extensions of the model followed by a brief conclusion.

I. Money-in-the-Utility Function Model

We set up a general equilibrium model, which is a generalization of the money in the utility function model in Lucas (2000). The model has a utility maximizing representative household and two profit maximizing representative firms. Households derive utility from the real stock of currency and the quantities of consumer goods and financial services. One firm produces consumer goods and the other firm produces financial services. The prices of currency, consumer goods, and financial services are determined by market clearing conditions.
Utility Maximizing Household:

The household maximizes lifetime utility as a function of consumption goods, $c$, real currency, $m = H / p_c$, and real financial services, $d$, where $p_c$ is price of the consumption good, and $H$ is the nominal stock of currency. The household maximizes lifetime utility:

$$\int_{s=0}^{\infty} e^{-\phi s} V(c(s), m(s), d(s)) ds,$$

subject to a flow budget constraint

$$c(s) + \rho_d(s) \cdot d(s) + \delta \cdot k(s) + \pi(s) \cdot m(s) + \dot{m}(s) =$$

$$\tau(s) + \theta(s) + (r(s) + \delta) \cdot k(s) + w_i(s) \cdot n_i(s) + w_2(s) \cdot n_2(s)$$

The notation is as follows: $\rho_d(s)$ is the relative price of financial services, $k(s)$ is the stock of physical capital owned by the household, $\delta$ is the rate of depreciation of physical capital, $\pi(s) = \dot{p}_c(s) / p_c(s)$ is the rate of consumer goods price inflation, $\tau(s)$ is the real value of a transfer of currency to the household from the government, $\theta(s)$ is income from other sources, $(r(s) + \delta)$ is the real user cost of capital, $n_i(s)$ is the labor supplied to sector $i$ ($i = 1, 2$), $w_i(s)$ is the real wage in sector $i$ ($i = 1, 2$) all at instant $s$. We assume that the rate of population growth is zero for simplicity.

The household owns all of the capital stock of the economy and rents it to the two firms at the rental price, $(r(s) + \delta)$. The household may also supply labor to either of the two sectors. We assume that households inelastically supply one unit of labor, which can be divided between both sectors so that $n_1 + n_2 = 1$. If the wages are not equal in the two sectors then households will allocate all labor to the sector with the higher wage. Therefore, interior solutions require that the wages be equal in both sectors. We can define total labor income as $\omega(s) = w_1(s) \cdot n_1(s) + w_2(s) \cdot n_2(s) = \max\{w_1(s), w_2(s)\}$. Define a new state variable, $a = k + m$, and rewrite the flow budget constraint as follows:

$$\dot{a}(s) = (\tau(s) + \theta(s) + \omega(s)) + r(s) \cdot a(s) - c(s) - \rho_m(s) \cdot m(s) - \rho_d(s) \cdot d(s)$$

(1)

The variable $\rho_m(s) = r(s) + \pi(s)$ is the real user cost of currency in continuous time.
The decision is equivalent to a problem in which $d$ is the real stock of a single interest-bearing bank deposit. If the bank deposit earns nominal interest rate $r_d(s) + \pi(s)$, then the flow budget constraint for the household would be as follows:

$$c(s) + \delta \cdot k(s) + \pi(s) \cdot m(s) + \pi(s) \cdot d(s) + \dot{k}(s) + \dot{m}(s) + \dot{d}(s) =$$

$$\tau(s) + \theta(s) + \left( r(s) + \delta \right) \cdot k(s) + \left( r_d(s) + \pi(s) \right) \cdot d(s) + w_1(s) \cdot n_1(s) + w_2(s) \cdot n_2(s)$$

Define the state variable, $a = k + m + d$, the real user cost of deposits as $\rho_d(s) = (r(s) - r_d(s))$ and substitute into the flow budget constraint to produce (1).7

Assuming $V$ is differentiable and quasi-concave and the solution is interior, the solution to the optimal control problem can be derived from the following conditions:

$$V_1(c(s), m(s), d(s)) = \mu(s)$$
$$V_2(c(s), m(s), d(s)) = \mu(s) \cdot \rho_m(s)$$
$$V_3(c(s), m(s), d(s)) = \mu(s) \cdot \rho_d(s)$$

$$\lim_{s \to \infty} e^{-\sigma s} \mu(s) \cdot a(s)$$

The first three conditions are the intra-temporal optimality conditions, which imply that the marginal rates of substitution between pairs of goods equal the corresponding relative price ratios. The fourth condition is the inter-temporal optimality condition, and the final condition is transversality. The variable $\mu$ is the costate variable for the current-value Hamiltonian.

**Profit Maximizing Goods Producers:**

Goods producing firms hire labor and rent capital to maximize instantaneous profits. Let $n_1(s)$ denote the labor hired in the goods producing sector, and $k_1(s)$ be capital rented by the goods producing firm at instant $s$. Goods are produced from the production function $y(s) = \lambda_1 \cdot f(n_1(s), k_1(s))$. The firm maximizes instantaneous profits:

$$\lambda_1 \cdot f(n_1(s), k_1(s)) - w_1(s) \cdot n_1(s) - \left( r(s) + \delta \right) \cdot k_1(s).$$

The necessary conditions for optimality are as follows:

$$\lambda_1 \cdot f_1(n_1(s), k_1(s)) = w_1(s), \quad \lambda_1 \cdot f_2(n_1(s), k_1(s)) = r(s) + \delta.$$
The instantaneous profits from the goods producing sector are paid to the household sector as income \( \theta_i(s) \). We assume that \( \lambda_i \) is constant for simplicity.

Profit Maximizing Financial Service Producers:

The financial service producing firm hires labor, \( n_z(s) \), and rents capital, \( k_z(s) \), to produce financial services. We assume labor and capital are transformed into financial services by the production function: \( d = \lambda_z \cdot h(n_z, k_z) \). This is the approach taken by King and Plosser (1984). The firm maximizes instantaneous profits:

\[
\rho_d \cdot \lambda_z \cdot h(n_z(s), k_z(s)) - w_z(s) \cdot n_z(s) - (r(s) + \delta) \cdot k_z(s).
\]

The necessary conditions for optimality are as follows:

\[
\rho_d(s) \cdot \lambda_z \cdot h_1(n_z(s), k_z(s)) = w_z(s), \quad \rho_d(s) \cdot \lambda_z \cdot h_2(n_z(s), k_z(s)) = r(s) + \delta
\]

The instantaneous profits from the goods producing sector are paid to the household sector as income \( \theta_z(s) \). Non-wage household income equals \( \theta(s) = \theta_i(s) + \theta_z(s) \). We assume that \( \lambda_z \) is constant for simplicity.

We consider some possible generalizations of this model in Section III. In particular, we comment on the possible implications of required reserves and excess reserves.

General Equilibrium Conditions in Steady State:

The optimality conditions imply that \( r^* = \phi \) in steady state. We assume that \( H(s)/p^*(s) = \tau(s) = \sigma \cdot m(s) \), and by definition, the growth rate of real currency is \( \dot{m}(s)/m(s) = H(s)/H(s) - \dot{p}_c(s)/p_c(s) = \sigma - \pi(s) \). Thus, the following additional conditions hold in steady state:

\[
\pi^* = \sigma, \quad \rho_m^* = \phi + \sigma.
\]

If \( n_1, n_2 \neq 0 \) then the wage in the two sectors must be equal. The conditions for profit maximization and wage equalization in steady state yield the conditions:

\[
\lambda_i \cdot f_1(n^*_i, k^*_i) = \rho_d \cdot \lambda_z \cdot h_1(n^*_z, k^*_z) = w^*, \quad \lambda_i \cdot f_2(n^*_i, k^*_i) = \rho_d \cdot \lambda_z \cdot h_2(n^*_z, k^*_z) = \phi + \delta.
\]

The aggregate resource constraints for the two types of goods and labor supply in steady state yield the conditions:

\[
c^* + \delta \cdot k^*_1 + \delta \cdot k^*_2 = \lambda_i \cdot f(n^*_i, k^*_i), \quad d^* = \lambda_z \cdot h(n^*_z, k^*_z), \quad n^*_1 + n^*_2 = 1.
\]
We assume that both production functions, \( f \) and \( h \), are homogeneous of degree one in capital and labor. This allows us to rewrite the steady state conditions in terms of the ratio of capital to labor, \( z_i^* = k_i^* / n_i^* \), for \( i = 1,2 \).

\[
\lambda_1 \cdot f_1 (1, z_1^*) = \rho_d^* \cdot h_1 (1, z_2^*) \quad (2)
\]

\[
\lambda_1 \cdot f_2 (1, z_1^*) = \phi + \delta \quad (3)
\]

\[
\rho_d^* \cdot \lambda_2 \cdot h_2 (1, z_2^*) = \phi + \delta \quad (4)
\]

\[
n_1^* + n_2^* = 1 \quad (5)
\]

\[
c^* + \delta \cdot n_1^* \cdot z_1^* + \delta \cdot n_2^* \cdot z_2^* = n_1^* \cdot \lambda_1 \cdot f (1, z_1^*) \quad (6)
\]

\[
d^* = n_2^* \cdot \lambda_2 \cdot h (1, z_2^*) \quad (7)
\]

The profits from both sectors will be zero in equilibrium because \( f \) and \( h \) are homogeneous of degree one in capital and labor.\(^8\)

The remaining household optimality conditions evaluated in steady state are as follows:

\[
V_1 (c^*, m^*, d^*) = \mu^* \quad (8)
\]

\[
V_2 (c^*, m^*, d^*) = \mu^* \cdot (\phi + \sigma) \quad (9)
\]

\[
V_3 (c^*, m^*, d^*) = \mu^* \cdot \rho_d^* \quad (10)
\]

Equations (2) - (10) determine the steady state values of \( (c^*, m^*, d^*, \mu^*, n_1^*, n_2^*, z_1^*, z_2^*, \rho_d^*) \).

**Impact of the Rate of Inflation on Steady State:**

We can make some general statements about the impact of inflation on the variables \( (c^*, m^*, d^*, \mu^*, n_1^*, n_2^*, z_1^*, z_2^*, \rho_d^*) \) by making several weak assumptions. We have assumed that production is constant returns to scale in both sectors. Under this assumption, it is easily seen that the variables \( z_1^* \), \( z_2^* \) and \( \rho_d^* \) are invariant to the rate of inflation in steady state, \( \sigma \). The value of \( z_1^* \) is determined by equation (3), and the values of \( z_2^* \) and \( \rho_d^* \) are determined by equations (2) and (4) given \( z_1^* \). We substitute equation (5) into equation (6) to get the following equation

\[
c^* = (1 - n_2^*) \cdot (\lambda_1 \cdot f (1, z_1^*) - \delta \cdot z_1^* + \delta \cdot z_2^*) - \delta \cdot z_2^* \quad (11)
\]
We can determine the impact of inflation on the remaining variables \((c^*, m^*, d^*, \mu^*, n_1^*, n_2^*)\) by differentiating (7) - (11) with respect to the rate of inflation in steady state, \(\sigma\), to produce the following system of comparative static-type equations:

\[
\begin{bmatrix}
0 & \lambda_2 \cdot h(1, z_2^*) & 0 & 0 & -1 & \frac{\partial \mu^*}{\partial \sigma} \\
0 & \lambda_1 \cdot f(1, z_1^*) - \delta(z_1^* - z_2^*) & 1 & 0 & 0 & \frac{\partial n_2^*}{\partial \sigma} \\
-1 & 0 & V_{11}^* & V_{12}^* & V_{13}^* & \frac{\partial c^*}{\partial \sigma} \\
-(\phi + \sigma) & 0 & V_{21}^* & V_{22}^* & V_{23}^* & \frac{\partial m^*}{\partial \sigma} \\
-\rho^*_d & 0 & V_{31}^* & V_{32}^* & V_{33}^* & \frac{\partial d^*}{\partial \sigma}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

where \(V_{ij}^* = V_{ij}(c^*, m^*, d^*)\). The sign of \(\frac{\partial c^*}{\partial \sigma}\) and \(\frac{\partial d^*}{\partial \sigma}\) is determined by \(\frac{\partial n_2^*}{\partial \sigma}\). Thus, the qualitative impact of \(\sigma\) on the steady state is determined by \(\frac{\partial m^*}{\partial \sigma}\) and \(\frac{\partial n_2^*}{\partial \sigma}\). The solution to this set of equations implies that

\[
\frac{\partial m^*}{\partial \sigma} = \frac{\mu^*}{|A|} \left( \lambda_1 \cdot f(1, z_1^*) - \delta \cdot z_1^* + \delta \cdot z_2^* \right) \cdot (V_{31}^* - \rho_d^* \cdot V_{11})
\]

(12)

\[
-\frac{\mu^*}{|A|} \lambda_2 \cdot h(1, z_2^*) \cdot (V_{33}^* - \rho_d^* \cdot V_{13})
\]

\[
\frac{\partial n_2^*}{\partial \sigma} = \frac{\mu^*}{|A|} (V_{32}^* - \rho_d^* \cdot V_{12})
\]

(13)

In appendix A.1, we show that \(|A|<0\) if \(c, d, m\) are all normal goods.

We can provide an economic interpretation of equation (12). Equations (8) and (10) imply that \(V_3 / V_1 = \rho_d^*\) in steady state, meaning that the marginal rate of substitution between the consumption good and financial services equals the relative price ratio. If we assume that \(V_3 / V_1\) is positively related to \(c^*\) and inversely related to \(d^*\), then \((V_{31}^* - \rho_d^* \cdot V_{11}) > 0\), \((V_{33}^* - \rho_d^* \cdot V_{13}) < 0\), and \(\frac{\partial m^*}{\partial \sigma} < 0\). Thus, an increase in the user cost of currency, which is caused by an increase in inflation, will cause real demand for currency to decrease in steady state. A formal result can be obtained if we assume that currency and financial services are weakly separable from consumption goods. Suppose there exists a sub-utility function, \(g\), such that the instantaneous utility function can be written as \(V(c, m, d) = U(c, g(m, d))\). If the
category sub-utility function, \( g \), is homothetic then it is an aggregator function and two stage budgeting theory is available under certain regularity conditions. If \( U \) is quasi-convex in \( c \) and \( g \), and \( c \) and \( g \) are both normal goods then \( \partial m^* / \partial \sigma < 0 \). We provide a formal proof and interpretation in appendix A.2.

We can also provide an economic interpretation of equation (13). A change in \( \sigma \) can cause a change \( m^* \), but not \( \rho_d^* \). If the change in \( m^* \) alters the marginal rate of substitution between the consumption good and financial services, \( V_3 / V_1 \), then \( c^* \) and \( d^* \) will adjust to equate the marginal rate of substitution to the relative price ratio, \( \rho_d^* \), which is invariant to inflation. The change in \( c^* \) and \( d^* \) implies a corresponding change in \( n_1^* \) and \( n_2^* \). The derivatives \( \partial c^* / \partial \sigma \) and \( \partial d^* / \partial \sigma \) cannot be signed without making additional assumptions about preferences.

II. The Welfare Cost of Inflation

Lucas (2000) estimates the welfare cost of inflation in a money-in-the-utility function model that is a special case of our model. In the Lucas model, non-interest bearing currency is the only type of money available to the household. This model coincides with our model in the special case \( \lambda_2 = 0 \), which implies that \( n_2^* = 0 \), \( c^* = \lambda_1 \cdot f(1,z_1^*) - \delta \cdot z_1^* \), and \( \lambda_1 \cdot f_2 (1,z_1^*) = (\phi + \delta) \). In this case, steady state consumption is invariant to \( \sigma \), and the real stock of currency in steady state is determined by the equation:

\[
\frac{V_2(c^*, m^*, 0)}{V_1(c^*, m^*, 0)} = (\phi + \sigma)
\]

(14)

The welfare cost of inflation can be measured by determining the proportion by which steady state consumption would have to increase to offset the loss in utility caused by inflation, relative to some benchmark level of inflation, \( \sigma^B \). Mathematically, we can view \( m^* \) as a function of \( \sigma \), and compute \( \lambda(\sigma, \sigma^B) \), such that the following equation is satisfied:

\[
U \left( c^* \cdot (1 + \lambda(\sigma, \sigma^B)), m^*(\sigma), 0 \right) = U \left( c^*, m^*(\sigma^B), 0 \right).
\]

The optimum rate of inflation is given by the Friedman rule, \( \sigma^{FR} = -\phi \), which leads to satiation in real currency. Typically, we would choose \( \sigma^B \) as either \( \sigma^{FR} \), a value close to \( \sigma^{FR} \) if
currency demand is not bounded, or zero. Lucas (2000) provides estimates of $\lambda(\sigma, \sigma^B)$ for the United States in a calibrated version of this model.\(^{10}\)

The welfare cost estimate can also be computed in our generalized model. In the generalized model, $c^*$ is not typically invariant to $\sigma$. We can view $c^*$, $m^*$, and $d^*$ as functions of $\sigma$, and compute $\lambda(\sigma, \sigma^B)$, such that the following equation is satisfied:

$$U\left(c^*(\sigma) \cdot \left(1 + \lambda(\sigma, \sigma^B)\right), m^*(\sigma), d^*(\sigma)\right) = U\left(c^*(\sigma^B), m^*(\sigma^B), d^*(\sigma^B)\right).$$

In order to compute $\lambda(\sigma, \sigma^B)$, we need to choose parametric functions for the utility and production functions and calibrate the values of the parameters.

A preferred concept of welfare cost, in terms of microeconomics, would be the concept of equivalent variations, EV. The definition of EV in this context equals

$$EV(\sigma, \sigma^B) = \min\left\{c + g \cdot \rho_g^*(\sigma) : U(c, g) = U\left(c^*(\sigma^B), g^*(\sigma)\right)\right\} - \left(c^*(\sigma) + g^*(\sigma) \cdot \rho_{g}^*(\sigma)\right).$$

EV is the minimum increase in real consumer expenditure on both goods and money at actual relative prices such that indirect utility is increased from $U\left(c^*(\sigma), g^*(\sigma)\right)$ to $U\left(c^*(\sigma^B), g^*(\sigma^B)\right)$, and it follows from the definition that $\lambda(\sigma, \sigma^B) \cdot c^*(\sigma) \geq EV(\sigma, \sigma^B)$. We calculated both $\lambda(\sigma, \sigma^B)$ and $EV(\sigma, \sigma^B)$ for all models, and found that the numerical differences are very small. We report only the $\lambda(\sigma, \sigma^B)$ forms of the estimates, because they are comparable with those in Lucas (2000).

Functional Forms:

We assume that currency and financial services are homotheticly weakly separable from consumer goods, so that $V(c, m, d) = U(c, g(m, d))$, where $g$ is a linearly homogeneous sub-utility function.\(^{11}\) Homothetic weak separability implies that $g$ evaluated at the optimum is a quantity aggregate. If financial services are bank deposits, then it is a monetary quantity aggregate. The dual price of this quantity aggregate is the unit expenditure function:

$$\rho_g(s) = e(\rho_m(s), \rho_d(s), 1) = \min\left\{h_m \cdot \rho_m(s) + h_d \cdot \rho_d(s) : g(h_m, h_d) = 1\right\}.$$ 

We assume that the quantity aggregate is CES:

$$g(m, d) = \left(m^h + a \cdot d^b\right)^{1/b},$$ (15)

where $b < 1$, and non-zero. The dual price aggregate has a dual CES form:
\[ \rho_g = \left( \rho_m^{\frac{b}{b-1}} + \psi \cdot \rho_d^{\frac{b}{b-1}} \right)^{\frac{1}{b-1}} \]  

(16)

where \( \psi = a^{1/(b-1)} \). This functional form is attractive because it strictly generalizes money-in-the-utility function models that have only non-interest bearing currency in the following sense: if \( \psi = 0 \), then \( d = 0 \), \( g(m, d) = m \), and \( \rho_g = \rho_m \).

Two stage budgeting theory is applicable under this assumption. The household can be viewed as making a two-stage decision.\(^{12}\) In the first stage, the household maximizes lifetime utility:

\[ \int_{s=0}^{\infty} e^{-\psi(s)} U(c(s), g(s)) ds, \]

subject to a flow budget constraint

\[ \dot{a}(s) = \left( \tau(s) + \psi(s) + \omega(s) \right) + r(s) \cdot a(s) - c(s) - \rho_g(s) \cdot g(s). \]

The solution to the first stage decision gives optimal values for the quantity of the aggregate good, \( g^*(s) \), given its relative price, \( \rho_g(s) \). The expenditure on the aggregate good is \( \gamma(s) = \rho_g(s) \cdot g^*(s) = e(\rho_m(s), \rho_d(s), g^*(s)) \), where the equality results from the linear homogeneity of \( g \). The second stage decision is to minimize total expenditure on \( m \) and \( d \), at each instant \( s \), subject to a constraint on the value of \( g \):

\[ \min_{m,d} \left\{ \rho_m(s) \cdot m + \rho_d(s) \cdot d : g(m, d) = g^*(s) \right\}. \]

Under our assumptions, the two-stage decision will have the same optimal solution as the original decision for all variables.

Assumptions about \( U \) will determine the relationship between \( c \) and \( g \) on the optimal path.\(^{13}\) Following Lucas (2000), we assume that \( U \) has a CRRA form:

\[ U(c, g) = \gamma^{-1} \cdot \left( c \cdot \Phi(g/c) \right)^\gamma, \]

where \( \Phi(z) = \left( 1 + z \cdot (A/z)^{1/\eta} \right)^{\eta/(\eta-1)} \). The value of \( \gamma \) does not have any effect on steady state or on our estimates of the welfare cost of inflation.\(^{14}\) The optimal control solution obeys the relationship \( g(s) = A \cdot c(s) \cdot \left( \rho_g(s) \right)^{-\eta} \), which is a log-log “money demand function” of the form:

\[ \ln(g/c) = \ln(A) - \eta \ln(\rho_g). \]  

(17)
We also assume that production in both sectors is Cobb-Douglas. The production functions have the form:

\[ y = \lambda_1 \cdot n_1 \cdot f(l, z_1) = \lambda_1 \cdot n_1 \cdot z_1^{1-\alpha} \]

\[ d = \lambda_2 \cdot n_2 \cdot h(l, z_2) = \lambda_2 \cdot n_2 \cdot z_2^{1-\beta} \]

**Steady State:**

We can solve for the steady state values \((c^*, m^*, d^*, n^*_1, n^*_2, z^*_1, z^*_2, \rho^*_d)\) under these assumptions:

\[ z^*_1 = \left( \frac{\lambda_1 (1-\alpha)}{\phi + \delta} \right)^{1/\alpha}, \quad z^*_2 = \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \]

\[ \rho^*_m = \phi + \sigma, \quad \rho^*_d = \left( \frac{z^*_2}{\lambda_2 (1-\beta)} \right)^\beta \]

\[ \rho^*_g = \left( \rho^*_m \right)^{b/(b-1)} + \psi \left( \rho^*_d \right)^{b/(b-1)/b} \]

\[ c^* = \frac{(\lambda_1 \cdot (z^*_1)^{1-\alpha} - \delta \cdot z^*_1) \cdot \lambda_2 \cdot (z^*_2)^{1-\beta}}{A \cdot \left( \rho^*_g \right)^{-\eta-1/(b-1)} \cdot \psi \cdot \left( \rho^*_d \right)^{1/(b-1)} \cdot (\lambda_1 \cdot (z^*_1)^{1-\alpha} - \delta \cdot z^*_1 + \delta \cdot z^*_2)} - \lambda_2 \cdot (z^*_2)^{1-\beta} \]

\[ m^* = A \cdot c^* \cdot \left( \rho^*_g \right)^{-\eta-1/(b-1)} \cdot \left( \rho^*_m \right)^{1/(b-1)} \]

\[ d^* = A \cdot c^* \cdot \left( \rho^*_g \right)^{-\eta-1/(b-1)} \cdot \psi \cdot \left( \rho^*_d \right)^{1/(b-1)} \]

\[ g^* = A \cdot c^* \cdot \left( \rho^*_g \right)^{-\eta} \]

\[ n^*_2 = \frac{d^*}{\lambda_2 \cdot (z^*_2)^{1-\beta}}, \quad n^*_1 = 1 - n^*_2 \]

The direction of the effect of \(\sigma\) on \(c^*\) is determined by the sign of \(\frac{1}{1-b} - \eta\), which is the elasticity of substitution between currency and financial services minus the own price elasticity of money demand. If currency and financial services are not substitutable then the elasticity of substitution between them is zero. In this case, an increase in \(\sigma\) would cause \(m^*\) and \(d^*\) to both fall, and this leads to reallocation of labor toward the goods producing sector from the financial services producing sector and therefore to an increase in \(c^*\). As the elasticity of substitution
between currency and financial services increases the negative effect of $\sigma$ on $d^*$ diminishes. If the elasticity of substitution is high enough that the difference between the elasticity of substitution and $\eta$ is positive, then the effect of $\sigma$ on $c^*$ and $d^*$ is reversed.\textsuperscript{16}

Calibration:

In order to use the model to produce an estimate of $\lambda(\sigma, \sigma^B)$, we need to calibrate the model. This involves choosing values for the following parameters: $\lambda_1, \lambda_2, \alpha, \beta, \delta, \phi, \sigma, \psi, b, A, \eta$. The parameter $\lambda_1$ defines the units for measurement of real consumption, and we set it equal to 1. Cooley and Prescott (1995) provide parameter values that can be converted to our continuous time parameters: $\alpha = .6, \delta = .079, \phi = .0545$.\textsuperscript{17} These values insure that the ratio of consumption to output and the ratio of capital stock to output are approximately equal to their average post-war values.\textsuperscript{18} The rate of currency growth, $\sigma$, is set equal to the average annualized rate of consumer price inflation in the United States during the period 1959-2000, $\sigma = .0411$.\textsuperscript{19}

We set $\psi$ and $b$ equal to the maximum likelihood estimates of the parameters of a homothetic CES second-stage demand system. The second-stage decision that generates the CES demand system does not depend on other parameters of the model, because of homothetic weak separability. The share equations for the demand system are as follows:

$$\hat{s}_m = \frac{\rho_m^{b/(b-1)} + \rho_d^{b/(b-1)} \cdot \psi}{\rho_m^{b/(b-1)} + \rho_d^{b/(b-1)} \cdot \psi}, \quad \hat{s}_d = \frac{\rho_d^{b/(b-1)} \cdot \psi}{\rho_m^{b/(b-1)} + \rho_d^{b/(b-1)} \cdot \psi}$$

We estimate $\psi$ and $b$ by maximum likelihood. We assumed that the true expenditure shares were generated by $s_{m,t} = \hat{s}_{m,t} + u_t$, where $u_t = \rho \cdot u_{i,t} + \epsilon_i$ and $\epsilon_i \sim N(0, \sigma^2)$.\textsuperscript{20} We assume a distribution around $u_t$ that ensures stationarity.\textsuperscript{21}

We obtained data for user costs and asset stocks of the deposit components of various monetary aggregates from the Federal Reserve Bank of St. Louis. Let $(d_1, ..., d_n)$ be a vector of real asset stocks of $n$ different types of deposits, and let $(\rho_{d1}, ..., \rho_{dn})$ be a vector of $n$ real user costs. We assume that the instantaneous utility function has the form:

$$V(c, m, d_1, ..., d_n) = U(c, g(m, d_1, ..., d_n)))$$
The function $d$ is a deposit aggregator function and $d(d_1,\ldots,d_n)$ evaluated at the optimum is a deposit quantity aggregate. The deposit price aggregate is the unit expenditure function, $\rho_d = e(\rho_{d1},\ldots,\rho_{dn},1)$. We compute superlative quantity and price index numbers from deposit stock and user cost data available online at the Federal Reserve Bank of St. Louis at various levels of aggregation for 1959-2000. We compute separate estimates at the M1a, MZM, and M2 levels of aggregation. In full generality, we could have included many types of deposits as separate goods in the utility function, although this would complicate the calibration of the model significantly.

We set $\eta$ equal to an econometric estimate of the parameter using data from the Federal Reserve Bank of St. Louis. There are two methods of estimating $\eta$. The first method is to regress $\ln(g/c)$ against a constant and $\ln(\rho_g)$. The negative of the slope is an estimate of $\eta$. We use superlative quantity index numbers to estimate $g$ and $c$, and we use a superlative price index number to estimate $\rho_g$ at the M1a, MZM, and M2 levels of aggregation for 1959-2000. The problem with this method is that the residual of the regression equation has very significant serial correlation and the variables appear to contain unit roots. We could treat $\eta$ as the coefficient in a co-integration relation between $\ln(g/c)$ and $\ln(\rho_g)$, but we were unable to find any evidence to support co-integration for our data. The serial correlation leads to large sustained errors during certain periods, particularly the 1990’s. An alternative approach is to employ the concept of a long run derivative. The optimal control relation in equation (17) has first difference form: $\Delta \ln(g/c) = -\eta \Delta \ln(\rho_g)$. We enrich (17) with lags to correct for serial correlation:

$$\Delta \ln(g/c)_t + \sum_{i=1}^n a_i \Delta \ln(g/c)_{t-i} = \sum_{i=0}^n b_i \Delta \ln(\rho_g)_{t-i} + \epsilon_t$$

and compute the long-run derivative, LRD, equal to

$$\text{LRD} = \frac{\partial \Delta \ln(g/c)}{\partial \Delta \ln(\rho_g)} = \frac{\sum_{i=0}^n b_i}{1 + \sum_{i=1}^n a_i}$$

from Fisher and Seater (1993). We set the lag length using the AI criterion. The LRD is an estimate of $-\eta$. The residuals from (19) are not unusual in the 1990’s and the LRD is reasonably robust to the sample period.
We calibrate $A$ so that the share $s_g = \frac{g(m^*, d^*) \cdot \rho^*}{\frac{1}{2} \left( g(m^*, d^*) \cdot \rho^* + c^* \right)}$ is equal to its average value for the superlative indexes during 1959-2000.\textsuperscript{27}

The parameter $\lambda_2$, which determines the units of financial services, is calibrated so that the steady state expenditure share of currency equals its average value during 1959-2000.\textsuperscript{28} The remaining parameter $\beta$ is set to equal $\alpha$.

We can also calibrate the model under the special case that all money is non-interest bearing currency. The special case is achieved by setting $\lambda_2 = 0$, and $\psi = 0$. The steady state values for the model are then $n_1^* = 1$, $n_2^* = 0$, $d^* = 0$, $\rho_m^* = \phi + \sigma$, $m^* = A \cdot c^* \cdot (\rho_m^*)^{-\eta}$, $z_1^* = \left( \frac{\lambda_1 \left( 1 - \alpha \right)}{\phi + \delta} \right)^{1/\alpha}$, and $c^* = \lambda_1 \cdot (z_1^*)^{1-\alpha} - \delta z_1^*$. The parameters $\lambda_1, \alpha, \delta, \phi$, and $\sigma$ are calibrated as described above. We use the estimate of $\eta$ corresponding to the level of aggregation from above, and we calibrate $A$ so that the expenditure share on $g$ is the same as for the generalized model at each level of aggregation.\textsuperscript{29}

The calibrated parameter values that are the same for all cases are given in Table 1A, the calibrated parameter values that change for each case are given in Table 1B. The steady state solutions are given in Table 2 for all cases.

Welfare Cost Estimates:

We solve for the welfare cost measure $\lambda(\sigma, \sigma^B)$ by bracketing the root of the function:

$$F(\lambda) = U \left( c^* (\sigma) \cdot (1 + \lambda), m^* (\sigma), d^* (\sigma) \right) - U \left( c^* (\sigma^B), m^* (\sigma^B), d^* (\sigma^B) \right)$$

and then we solve for the root numerically. We calculate $\lambda(\sigma, \sigma^B)$ with $\sigma^B = -0.054$ and $\sigma \in (\sigma^B, 0.2)$ using a numerical procedure.\textsuperscript{30} We give some representative values for $100 \cdot \lambda(\sigma, \sigma^B)$ in Table 3, but the results are easier to understand graphically. We therefore provide graphs of the welfare cost estimates. Graph 1 contains estimates at the M1a level, Graph 2 at the MZM level, and Graph 3 at the M2 level. We also estimated the equivalent variation measure of welfare cost described above and found that the differences were not numerically important.\textsuperscript{31}

Main Conclusions:

Several main conclusions are evident:
i) The welfare cost of inflation in the generalized model is very low when compared with the estimates in Lucas (2000), which closely resemble case 13. In that case, we found that the benefit from reducing the steady state rate of inflation from 20% to −5% would be equivalent to an increase in steady state consumption of almost 2.5%. The estimates from cases 1-6 provide much more modest estimates, the increase in steady state consumption range from as low as .207% to as high as .59%.

ii) The welfare cost of inflation is always much lower in the models with non-interest bearing currency and interest bearing deposits than in the models with only non-interest bearing currency. We investigated a variety of different cases corresponding to different levels of aggregation and different methods for calibrating the crucial parameter $\eta$. Cases 1-6 correspond to the generalized model and these are paired with cases 7-12 in which only non-interest bearing currency is held. The welfare cost of inflation is much less in all cases for the models in which interest-bearing deposits can be held versus the models with only non-interest bearing currency. The basic reason is that the larger the expenditure share of interest bearing deposits versus non-interest bearing currency the less the aggregate price of money $g_\rho$ increases in response to an increase in inflation. The smaller the effect of $\sigma$ on $g_\rho$ the weaker the desire to substitute away from aggregate money (in other words, the weaker the effect on $g^*/c^*$) and consequently the lower the welfare cost of inflation. For our calibrations, the effect of increases in $\sigma$ on steady state consumption, $c^*$, are positive but very small.

iii) The welfare cost of inflation is positively related to the elasticity of the aggregate money demand relation, $\eta$, which can be seen by comparing cases 1-3 to cases 4-6. The logic behind this is that the higher the value of $\eta$, the more $g^*/c^*$ will need to adjust in response to a given change in $\sigma$. In our opinion, cases 4-6 produce more plausible estimates, because of econometric considerations. The linear regression that is used to produce estimates of the own price elasticity of money demand in cases 1-3 suffers from serial correlation problems, whereas the LRD that is used in cases 4-6 is based on an estimated autoregression. Nevertheless, our basic conclusion that the welfare cost of inflation is quite low is robust to the range of estimates in cases 1-6.
III. Extensions of the Model

The model presented in this paper can be enriched in several ways. First, we have assumed that all deposits (including demand deposits) are interest bearing. We could generalize this by allowing the banking system to produce multiple deposit types, some of which are non-interest bearing. This would be expected to raise the estimate of the welfare cost of inflation, because it would raise the expenditure share on the non-interest bearing monetary components. Nevertheless, all types of deposits in conventional monetary aggregates are interest bearing except for demand deposits and there is substantial empirical evidence that the implicit rate of interest on demand deposits is positive.

Second, we have assumed that all interest-bearing deposits are weakly separable from non-interest bearing currency. We could generalize the model to include deposits that are not weakly separable. The cost of doing this is that it could reduce the precision of our calibration by adding numerous additional preference and technology parameters. We could also test explicitly for weakly separable groupings of deposits using available econometric techniques.34

Third, we could consider more general specifications of the optimal control relationship between the aggregate quantity of money, the aggregate price of money, and real consumption. We considered the class of utility functions that produce “money demand functions” that exhibit unit consumption elasticity, which does not seem to be strongly supported by the data. We could generalize the model to allow the consumption elasticity to differ from one.

Fourth, we could calibrate an extended version of either the shopping time or cash-in-advance models. The shopping time model has the advantage that the welfare cost of inflation is clearly defined in terms of lost production due to time spent shopping. If the rate of inflation increases, the household will spend more time shopping to economize on non-interest bearing currency. We expect that the basic finding of this paper will remain valid. The existence of interest-bearing assets that reduce shopping time will lead to substantially lower estimates of the cost of inflation.

Finally, we could enrich the supply side of the model to include multiple inputs and outputs and reserves, following work such as Hancock (1985,1986), Barnett (1987), and Fixler and Zieschang (1999). If financial services producing firms must hold required reserves then the rate of inflation will enter the profit function for those firms. Suppose that each unit of deposits
must be backed by $x$ units of required reserves, where $0 < x < 1$. Each unit of required reserves impose a tax equal to the user cost of currency, so that the profit function becomes:

$$\rho_d - x \rho_m \cdot \lambda_2 \cdot h(n_2(s), k_2(s)) - w_2(s) \cdot n_2(s) - (r(s) + \delta) \cdot k_2(s)$$

The supply price of deposits, $\rho_d^s = (\rho_d - x \rho_m)$, differs from the demand price of deposits, $\rho_d$, by an inflation tax. In our model, wages and user costs of capital are equalized across sectors. This would imply that $\rho_d^s$ will be invariant to $\sigma$ in steady state, but the demand price will satisfy $\frac{\partial \rho_d}{\partial \sigma} = x < \frac{\partial \rho_m}{\partial \sigma} = 1$. Thus, the user cost of interest-bearing deposits will be less sensitive to inflation than the user cost of non-interest bearing currency, but it will not be invariant to inflation in steady state. This implies that a model with required reserves should lead to higher estimates of the welfare cost of inflation than those presented here, though it will still be substantially lower than in a model with only non-interest bearing currency. Nevertheless, different types of deposits are subject to different required reserve rates and this will complicate the calibration and solution of the model, and will limit the usefulness of the aggregation-theoretic techniques we have made extensive use of in this paper. A more general possibility would be to incorporate excess reserves as a factor of production. This would be a much richer model, because the optimal level of labor and capital in the financial services sector would both be dependent on the level of inflation, but it is difficult to predict how this might alter our conclusions about the welfare cost of inflation.

**Conclusions**

We have generalized a standard money-in-the-utility function model to include both currency and interest bearing deposits. We calibrated the model for different levels of monetary aggregation using superlative monetary index numbers and estimate the welfare cost of inflation. We found that the expenditure share of non-interest bearing currency is relatively low compared to the expenditure share on interest-bearing deposits, and this implies that the welfare cost of inflation is substantially lower in this generalized model than in models in which money consists of only non-interest bearing currency. If we assume that MZM consists entirely of non-interest bearing currency and use a regression based estimate of the own price elasticity of money demand then the estimated benefit of lowering the rate of inflation from 20% to −5% would be equivalent to an increase in steady state consumption of almost 1.8% (using estimates based on the LRD). If we assume that MZM consists of both non-interest bearing currency and interest-
bearing deposits and use our generalized model then the estimated benefit of lowering the rate of inflation from 20% to −5% would be equivalent to an increase in steady state consumption of only .261%, which is very small.

The intuition for our result is that if only a small portion of the monetary portfolio is composed of non-interest bearing assets then an increase in the rate of inflation has a small effect on the aggregate price of money and therefore leads to comparatively small estimates of the welfare cost. If our estimates are believed, then this channel by which inflation leads to welfare loss is insignificant and the resources spent fighting inflation must be justified in some alternative way.
Appendix

A.1 Signing the determinant of A

We rewrite the system of equations here for the convenience of the reader.

\[
\begin{pmatrix}
0 & h(1, z_1^*) & 0 & 0 & -1 & \partial \mu^* / \partial \sigma \\
0 & f(1, z_1^*) - \delta z_1^* + \delta z_2^* & 1 & 0 & 0 & \partial n_1^* / \partial \sigma \\
-1 & 0 & V_{11}^* & V_{12}^* & V_{13}^* & \partial c^* / \partial \sigma \\
-(\phi + \sigma) & 0 & V_{21}^* & V_{22}^* & V_{23}^* & \partial m^* / \partial \sigma \\
-\rho^*_d & 0 & V_{31}^* & V_{32}^* & V_{33}^* & \partial d^* / \partial \sigma
\end{pmatrix}
\]

We get the following equation:

\[
|A| = \left( f(1, z_1^*) - \delta z_1^* + \delta z_2^* \right) \begin{vmatrix} -1 & V_{11} & V_{12} \\ -\phi + \sigma & V_{21} & V_{22} \\ -\rho^* & V_{31} & V_{32} \end{vmatrix} + h(1, z_2^*) \begin{vmatrix} -1 & V_{12} & V_{13} \\ -\phi + \sigma & V_{22} & V_{23} \\ -\rho^* & V_{32} & V_{33} \end{vmatrix}
\]

Consider the following static microeconomic problem:

\[
\max_{c,m,d} \{ V(c,m,d) : p_c \cdot c + p_m \cdot m + p_d \cdot d = y \}
\]

If you differentiate the first order conditions with respect to \( y \) and solve for \( \frac{\partial c}{\partial y} \) and \( \frac{\partial d}{\partial y} \), then

\[
\frac{\partial c}{\partial y} > 0 \text{ and } \frac{\partial d}{\partial y} > 0 \text{ if } \begin{vmatrix} -1 & V_{11} & V_{12} \\ -\phi + \sigma & V_{21} & V_{22} \\ -\rho^* & V_{31} & V_{32} \end{vmatrix} < 0 \text{ and } \begin{vmatrix} -1 & V_{12} & V_{13} \\ -\phi + \sigma & V_{22} & V_{23} \\ -\rho^* & V_{32} & V_{33} \end{vmatrix} < 0.
\]

This normal goods assumption implies that \( |A| < 0 \).

A.2 The derivative of steady state real money demand with respect to inflation

In the text of the paper, we showed that derivative of real currency in steady state with respect to the steady state inflation rate was given by the following:

\[
\frac{\partial m^*}{\partial \sigma} = \left( f(1, z_1^*) - \delta z_1^* + \delta z_2^* \right) \begin{vmatrix} V_{31} - \rho^*_d \cdot V_{11} \\ V_{32} - \rho^*_d \cdot V_{12} \\ V_{33} - \rho^*_d \cdot V_{13} \end{vmatrix}
\]

Now suppose that \( m \) and \( d \) are weakly separable in the utility function:

\[
V(c,m,d) = U(c, g(m,d))
\]

Weak separability implies that
\[ V_{31} = U_{11} \cdot g_2 \cdot V_{11} = U_{12} \cdot g_2, \quad V_{13} = U_{12} \cdot g_2, \quad V_{33} = U_2 \cdot g_2^2 + U_2 \cdot g_2 
\]

Optimization and weak separability imply that \( \rho_d^* = \frac{U_2}{U_1} \cdot g_2 \). Substitution yields the following equation:

\[
\frac{\partial m^*}{\partial \sigma} = \frac{\mu^*}{|A|} \left( f(1, z_1^*) - \delta \cdot z_1^* + \delta \cdot z_2^* \right) \cdot g_2 \cdot \left( U_{21} - \frac{U_2}{U_1} \cdot U_{11} \right)
\]

\[
-\frac{\mu^*}{|A|} \cdot h(1, z_2^*) \cdot g_2^2 \cdot \left( U_{22} - \frac{U_2}{U_1} \cdot U_{12} \right)
\]

\[
-\frac{\mu^*}{|A|} \cdot h(1, z_2^*) \cdot U_2 \cdot g_2
\]

Consider the following static microeconomic decision:

\[
\max_{c,m,d} \{ U(c, g) : p_c \cdot c + p_g \cdot g = y \}
\]

If you differentiate the first order conditions with respect to \( y \), and solve for \( \frac{\partial c}{\partial y} \) and \( \frac{\partial g}{\partial y} \), then quasi-concavity of \( U \) implies \( \frac{\partial c}{\partial y} > 0 \) and \( \frac{\partial g}{\partial y} > 0 \) if

\[
\left( U_{21} - \frac{U_2}{U_1} \cdot U_{11} \right) > 0, \quad \left( U_{22} - \frac{U_2}{U_1} \cdot U_{12} \right) < 0.
\]

These conditions and \( g_{22} < 0 \) imply that \( \frac{\partial m^*}{\partial \sigma} < 0 \). The interpretation is that \( \frac{\partial m^*}{\partial \sigma} < 0 \) if consumption and the aggregate good, \( g \), are both normal. The term, \( \frac{U_2}{U_1} \), is interpreted as the relative price of the aggregate good and is equal to \( \rho_g \) in equilibrium.
### Table 1A: Calibrated Parameter Values – All Cases

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### Table 1B: Calibrated Parameter Values – Specific Cases

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<th>Case</th>
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<th>$b$</th>
<th>$A$</th>
<th>$\eta$</th>
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</table>
| 1 - Calibrated to match data for M1a  
$\eta$ is from linear regression, $A$ calculated from expenditure share | 16.45 | 1.88 | -19.2 | 0.026 | 0.68 |
| 2 - Calibrated to match data for MZM  
$\eta$ is from linear regression, $A$ calculated from expenditure share | 16.62 | 5.97 | -6.39 | 0.0425 | 0.59 |
| 3 - Calibrated to match data for M2  
$\eta$ is from linear regression, $A$ calculated from expenditure share | 19.94 | 9.46 | -10.46 | 0.0615 | 0.33 |
| 4 - Calibrated to match data for M1a  
$\eta$ is -LRD, $A$ calculated from expenditure share | 16.45 | 1.88 | -19.2 | 0.0409 | 0.38 |
| 5 - Calibrated to match data for MZM  
$\eta$ is -LRD, $A$ calculated from expenditure share | 16.62 | 5.97 | -6.39 | 0.0526 | 0.15 |
| 6 - Calibrated to match data for M2  
$\eta$ is -LRD, $A$ calculated from expenditure share | 19.94 | 9.46 | -10.46 | 0.0654 | 0.15 |
<p>| 7 – $\eta$ from case 1, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.034 | 0.68 |
| 8 – $\eta$ from case 2, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.091 | 0.59 |
| 9 – $\eta$ from case 3, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.236 | 0.33 |
| 10 – $\eta$ from case 4, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.0688 | 0.38 |
| 11 – $\eta$ from case 5, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.2557 | 0.15 |
| 12 – $\eta$ from case 6, $A$ calibrated to equate expenditure share | 0 | 0 | N/A | 0.36 | 0.15 |
| 13 – parameters from Lucas (2000) | 0 | 0 | N/A | 0.05 | 0.5 |</p>
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Table 3: Welfare Cost of Inflation as a % of Steady State Consumption
Selected Estimates of 100 · \(λ(σ, σ_i^B)\) for Cases 1-13

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References


Barro, Robert, and Anthony Santomero. “Household Money Holdings and the Demand Deposit Rate” Journal of Money, Credit, and Banking May 1972, 397-413.


Endnotes

1 Lucas (2000) finds that the estimated welfare cost of inflation is numerically close to the area under the inverse money demand function, which establishes the connection to the classic literature going back to Bailey (1956).

2 See for examples, Barnett and Spindt (1982), Farr and Johnson (1985), Thornton and Yue (1992), and Anderson, Jones, and Nesmith (1997). See also, Barro and Santomero (1972), Klein (1975), Becker (1975), and Startz (1979). The Startz (1979) approach is the one taken by the most widely available data produced by the Federal Reserve Bank of St. Louis and described in Anderson, Jones, and Nesmith (1997).

3 Lucas (2000) also calculated the welfare cost of inflation in a shopping time model, with only non-interest bearing money. Simonsen and Cysne (2001) extend a shopping time model to include both currency and interest-bearing deposits in a shopping time model. They derive upper and lower bounds for the welfare cost of inflation assuming that the interest rate on deposits is invariant to inflation. The bound results have been extended under weak separability to allow the interest rate on deposits to vary with inflation. Simonsen and Cysne (2001) do not calibrate the model or provide estimates of the welfare cost.

4 We assume that demand deposits are implicitly interest bearing. If we instead assumed that these were non-interest bearing we would need to modify our model significantly, and it would lead to increased estimates of the welfare cost of inflation. We can therefore think of our results as being conservative estimates of the welfare cost of inflation and compare and contrast them with the much higher estimates provided by Lucas (2000).

5 The variable, \( r \), can be thought of as either the real interest rate or the user cost of capital net of depreciation.

6 The nominal user cost of capital is \( p(s) \cdot (r(s) + \delta) \), the nominal user cost of currency is \( p(s) \cdot (r(s) + \pi(s)) \), and the nominal price of financial services is \( p(s) \cdot \rho(s) \).

7 The nominal user cost of deposits is \( p(s) \cdot (r(s) - \rho(s)) \). This is a continuous time version of the formula derived in Barnett (1978).

8 Thus, \( \theta^* = 0 \) is an implied steady state condition.

9 This rate is optimal in the following sense: suppose that an agent maximizes \( \int_{-\infty}^{\infty} e^{-\phi(s)} V(c(s), m(s), d(s)) ds \) subject to the constraints: \( k + c + \delta k = \lambda_1 \cdot n_1 \cdot f(1, z_1) \), \( d = \lambda_2 \cdot n_2 \cdot h(1, z_2) \), and \( n_1 + n_2 = 1 \). The necessary conditions for optimality imply that \( V'(c, m, d) = 0 \).

10 There are some minor differences between the Lucas (2000) money-in-the-utility function model and our version of it.

11 We make extensive use of monetary aggregation theory in this section. If the reader is unfamiliar with this theory, consult Barnett (1987), Barnett and Serletis (2000), or Anderson, Jones, and Nesmith (1997).

12 See Blackorby, Primont, and Russell (1976) for precise discussions of decentralization and multi-stage budgeting.

13 The necessary conditions for optimality would imply that \( U_z(c, g) = \rho \). If \( U \) is linearly homogeneous then

\[
\zeta(g/c) = U_z(1, g/c) \quad \zeta(g/c) = \rho \quad \text{and monotonicity would imply that} \quad g = c \cdot \zeta^{-1}(\rho). 
\]

14 It is critical to the behavior of the model on the transition path, however.

15 Thus, \( g^*/c^* \) always decreases in response to an increase in steady state inflation. Let \( z = g/c \) then \( z^* = A(\rho^*)^{\eta} \).

16 We estimate both the elasticity of substitution between currency and deposits and the own price elasticity of money demand in the next section. We find that \( \frac{1}{1-b} - \eta \) is negative for the cases we investigate.

17 We get the parameter \( \delta \) from the ratio of investment to capital stock in steady state.

18 The ratio of consumption to output should be approximately .77, and the ratio of capital to output should be approximately 3.16. We use the same parameter values for \( \phi \) and \( \delta \) for all cases, but our model approximately satisfies these conditions in all cases. We could have made minor changes in the parameter values to satisfy these conditions exactly, but our main conclusions are robust to such changes.
The computations are: 
\[ \delta = -\ln(1-.076) = .079, \quad \phi = -\ln(.947) = .0545, \quad \text{and} \quad \sigma = \ln(1.0419) = .0411. \]

The rate of consumer price inflation is calculated from a superlative index of non-durable and services components of Personal Consumption Expenditures (PCE).

The expenditure shares satisfy the identity \( s_{m,j} + s_{d,j} = 1 \), so we use only \( s_{m,j} \) in our estimation. The choice of share to delete is arbitrary and does not affect the maximum likelihood estimator.

M1a is defined as currency, traveler’s checks, and demand deposits. The data we are using is based on the assumption that demand deposits earn implicit interest. M2 contains all components of M2 except small denomination time deposits.

We also note that for two good demand systems the CES utility function is a flexible functional form.

Superlative quantity and price index numbers for M1a, MZM, and M2 are available online from the Federal Reserve Bank of St. Louis. We created a superlative quantity index for the non-durable and service components of PCE.

See Pollack and Wales (1992) for details.

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Superlative quantity and price index numbers for M1a, MZM, and M2 are available online from the Federal Reserve Bank of St. Louis. We created a superlative quantity index for the non-durable and service components of PCE.

Lucas (2000) also notes this in his regressions.

The data appear to be I(1). The form of LRD depends on the order of integration.

The share \( s_{m,j} \) is equal to \( \frac{A(\rho^*_s)^{1-\eta}}{A(\rho^*_s)^{1-\eta} + 1} \) in steady state. The average expenditure share calculated using data from 1959-2000 is .0158 for M1a, .0336 for MZM, and .0467 for M2. The authors can provide detailed information underlying these calculations upon request.

The steady state expenditure share for currency in the second-stage decision is

\[ s_m = \frac{m^* \cdot \rho^*_m}{m^* \cdot \rho^*_m + d^* \cdot \rho^*_d} = \frac{1}{(1+\psi \cdot (\rho^*_d / \rho^*_m)^{(b-h-1)})} \]

The average expenditure share for currency from 1959-2000 is .45 for M1a, .2 for MZM, and .16 for M2. The relative price \( \rho^*_d / \rho^*_m = (\lambda^*_c \cdot (1-\beta))^{-1} \).

Alternatively, we could have assumed that the user costs of all monetary assets equal the nominal interest rate and recalibrated both parameters using real data. This is essentially the procedure used by Lucas (2000). This method would lead to increased aggregate expenditure on money, because the user cost of an interest-bearing asset is always lower than the nominal interest rate. This would lead to an increase in the estimated welfare cost of inflation for the non-interest bearing money cases. Our main finding is that the welfare cost is lower for the generalized model than for the non-interest bearing money cases, thus, we are being conservative in this sense. Case 13, which is equivalent to the estimates in Lucas (2000), is provided for purposes of comparison.

The Friedman rule inflation rate cannot be used as our benchmark, because the demand function for currency is not bounded if its user cost price is zero for the utility function being investigated.

For the functional forms investigated in this section, the expenditure function

\[ \min_{c,g} \left\{ c + g \cdot \rho^*_s(\sigma) : U(c,g) = U(c^*(\sigma^B), g^*(\sigma^B)) \right\} \]

is equal to the following:

\[ c^*(\sigma^B) \left(1 + A \cdot (\rho^*_s(\sigma)^{1-\eta}) \right) \left[ 1 + A \cdot (\rho^*_s(\sigma)^{1-\eta}) \right]^{-\eta/(\eta-1)} \]

The EV estimates are available from the authors upon request.

A change in the rate of steady state inflation \( \sigma \) effects the user cost of currency, \( \rho^*_m \), on a one-for-one basis. The user cost of deposits, \( \rho^*_d \), is invariant to \( \sigma \). Thus, the effect of \( \sigma \) on the aggregate price of money, \( \rho^*_s \), is summarized by the elasticity \( \frac{\partial \rho^*_s}{\partial \rho^*_m} \cdot \frac{\rho^*_m}{\rho^*_s} = (1-s_d) = s_m < 1 \). Thus, the larger the expenditure share on non-interest bearing currency (or equivalently the smaller the expenditure share on interest-bearing deposits) the larger the effect of a change in steady state inflation on the aggregate price of money in percentage terms.

This is because \( \frac{\partial z}{\partial \rho^*_s} \cdot \rho^*_s = -\eta, \) where \( z = g/c \).

Although, see Barnett and Choi (1989).
35 See Barnett, Hinich, and Weber (1986) for a discussion of this “price wedge”.