Total Factor Productivity and the Environmental Kuznets Curve:  
A Comment and Some Intuition

Neha Khanna*
Department of Economics
Binghamton University, P.O. Box 6000
Binghamton, NY 13902-6000
Phone: 607-777-2689, Fax: 607-777-2681, Email: nkhanna@bux2k.binghamton.edu

and

Florenz Plassmann
Department of Economics
Binghamton University, P.O. Box 6000
Binghamton, NY 13902-6000
Phone: 607-777-4304, Fax: 607-777-2681, Email: fplass@binghamton.edu

Department of Economics Working Paper 0518
This version: Sept 22, 2006.

* Corresponding author.
Abstract

Chimeli and Braden (2005) derive a necessary and sufficient condition under which inter-country differences in total factor productivity can yield an Environmental Kuznets Curve. They argue that their results emphasize the importance of differences in total factor productivity across countries as well as the need for decreasing returns to scale in pollution-abating technologies for the existence of an EKC. We show that their Proposition 1 is equivalent to Proposition 2 in Lieb (2002). This implies that, even in Chimeli and Braden’s model, contemporaneous changes in the marginal rate of substitution between environmental quality and consumption on the demand side and the marginal rate of transformation between these goods on the supply side drive the pollution-income relationship. This is a very general condition that does not rely on either differences in total factor productivity or decreasing returns to scale in abatement and is widely applicable.

Key words: abatement, environmental quality, pollution, marginal rate of substitution, marginal rate of transformation

JEL classification codes: Q50, Q56
1. Introduction

Chimeli and Braden (2005) show that differences in total factor productivity can yield a U-shaped relationship between environmental quality and income in a cross section of countries, a relationship implied by the Environmental Kuznets Curve (EKC) hypothesis. Several authors have empirically tested the EKC hypothesis using either cross-sectional data (for example Gawande et al., 2000, Khanna and Plassmann, 2004) or data that cover a relatively short time period (for example, Torras and Boyce, 1998), and Chimeli and Braden’s result provides a theoretical justification for these tests.

In their proposition 1, Chimeli and Braden derive a condition that is necessary and sufficient for a cross-sectional EKC under their model assumption. This condition is fairly complex, and they offer only a narrow explanation for their result that emphasizes differences in total factor productivity across countries as well as the need for decreasing returns to scale in pollution-abating technologies. We show that their necessary and sufficient condition is equivalent to Proposition 2 in Lieb (2002). We use this equivalence to derive an economically appealing and general interpretation for Chimeli and Braden’s result: that the existence of an EKC simply depends on appropriate joint changes in the marginal rates of substitution and transformation and differences in income or resources, regardless of what the source of those differences might be.

In the following section, we provide a brief outline of both models. We retain the original notations as far as possible for the sake of comparability with the original papers, but we compare and contrast the notations to emphasize a few subtle differences in the setup of the two models. We establish the equivalence of the two propositions in Section 3, and provide the economic intuition for Chimeli and Braden’s result in Section 4.
2. The Models

1) Chimeli and Braden (2005) (henceforth C&B):

Individual utility, $u$, is determined by the flow of per capita consumption, $c$, and the stock of environmental quality, $E$, and it is maximized subject to the equations of motion for $E$ and the capital stock, $K$. Both $E$ and $c$ are economic goods, so that $u_E > 0, u_c > 0$. Environmental quality decreases with gross anthropogenic pollution, $P$, and improves with pollution abatement, $\Pi$, which is a non-linear function of environmental protection effort, $\pi$. A social planner maximizes the present value of social welfare over an infinite time horizon. The economy’s output, $F$, is divided between aggregate consumption, environmental protection, and capital accumulation. The model is closed by specifying initial values for $E$ and $K$. The social planner’s problem is described formally as:

$$\max_{c, \pi} \int_0^\infty e^{-\rho t} N u(c, E) dt$$

subject to

$$\dot{E} = -P(K) + \Pi(\pi)$$

$$\dot{K} = F(K) - Nc - \pi$$

$$E, K, c, \pi \geq 0$$

$$E(0) = E_0, K(0) = K_0,$$

where $\rho > 0$ is the discount rate and $N$ is the population size. C&B specify assumptions about the curvature of the utility, gross pollution, output, and abatement functions so as to obtain a unique and interior solution to the social planner’s problem. Because this is an

---

1 To be consistent with C&B, we use the term pollution abatement in the broadest sense so that it includes not only pollution reduction and prevention, but also recovery as well as the direct generation of environmental quality through the creation of nature preserves, species protection, etc.

2 This is a generalized version of Problem 9 in Weitzman (2003, see pp. 56-60 and 173-181).
infinite horizon model, they evaluate the comparative statics that yield an EKC for a cross-section of economies at the steady state. It is in this context that their analysis is conceptually equivalent to the static, representative agent model developed by Lieb (2002; see also Weitzman, 2003, pp. 19-25), which, in turn, is a more general version of McConnell (1997) and Stokey (1998).


Individual utility, $U$, is a function of consumption, $C$, and net anthropogenic pollution, $P$. Consumption is an economic good and net pollution is an economic ‘bad,’ so that $U_C > 0, U_P < 0$. Net pollution increases with $C$ and decreases with abatement expenditures, $A$. Outlays on consumption and abatement come directly from the economy’s endowment of resources, $Y$. Thus the representative agent’s problem is

$$\max_{C,A} U(C,P)$$

subject to

$$P = P(C,A)$$
$$Y = C + A$$
$$A \geq 0, P \geq 0.$$

Lieb makes additional assumptions regarding the slope and curvature of the net pollution function ($P_C > 0, P_A < 0, P_{CC} \geq 0, P_{AA} > 0$, and $P_{CA} = P_{AC} \leq 0$), which imply that the consumption possibilities curve between $C$ and $P$ (which is defined by the net pollution function for given $Y$) is strictly convex.
3) Note the notation

The notation and terminology used in these two papers is sufficiently similar to obscure differences between the two models. We outline some of the main differences and potential sources of confusion. First, Lieb defines \( P \) as net pollution (gross pollution less abatement), whereas C&B define \( P \) as gross pollution. Second, Lieb's net pollution \( P \) is equivalent to the negative of C&B's environmental quality \( E \), under the assumption that environmental quality is the difference between the pristine state of the environment, \( O \), and anthropogenic net pollution, and normalizing \( O \) to zero. Third, Lieb's abatement expenditure, \( A \), is the same as C&B's \( \pi \). Fourth, C&B model pollution abatement, \( \Pi \), as a function of \( \pi \), and there is no equivalent to \( \Pi \) in Lieb (but that is irrelevant for our purposes). Finally, C&B use the variable \( A \) to denote total factor productivity. In a cross-section of economies in steady state, differences in \( A \) across countries describe differences in output. So \( dA \) represents a change in steady state output and is therefore equivalent to \( dY \) in Lieb.

3. The equivalence between C&B’s Proposition 1 and Lieb’s Proposition 2

1) Lieb:

Net pollution \( P \) is a function of consumption \( C \) and abatement \( A \), so that

\[
P_C = P_C(C, A) \quad \text{and} \quad P_A = P_A(C, A),
\]

and therefore

\[
dP_C = P_{CC}dC + P_{CA}dA \quad \text{and} \quad dP_A = P_{AC}dC + P_{AA}dA.
\]

Dividing the two total derivatives by \( dC \) and evaluating them at \( dP = 0 \) yields

\[
\left. \frac{dP_C}{dC} \right|_{dP=0} = P_{CC} + P_{CA} \left. \frac{dA}{dC} \right|_{dP=0} = V \quad \text{and} \quad \left. \frac{dP_A}{dC} \right|_{dP=0} = P_{AC} + P_{AA} \left. \frac{dA}{dC} \right|_{dP=0} = W.
\]

Denote the
marginal rate of substitution between consumption and pollution as \( MRSC^C.P = -\frac{U_C}{U_P} > 0 \).\(^3\)

The assumptions that consumption and environmental quality (negative pollution) are a normal goods and that pollution is a bad imply \( MRSC^C.P = \frac{\partial MRSC^C.P}{\partial C} < 0 \) (see Lieb, 2002, footnote 4).

Lieb’s Proposition 2 states that the direction of change in net pollution in response to a change in income depends on the relative magnitude of the change in the marginal rate of substitution with respect to \( C \) and the difference between \( W \) and \( V \), or

\[
\frac{dP}{dY} > 0 \iff -MRSC^C.P < W - V = \left. \frac{dP_A}{dC} \right|_{dP=0} - \left. \frac{dP_C}{dC} \right|_{dP=0}.
\]  

(3)

Expression (3) has a very straightforward economic interpretation. Efficiency requires \( MRSC^C.P = MRT^C.P \), where \( MRT^C.P \) is the slope of the consumption possibilities curve in \((C, P)\)-space. Net pollution depends on the levels of consumption and abatement, so that \( MRT^C.P = P_c - P_A \).\(^4\) The total derivative of \( MRT^C.P \) is \( dMRT^C.P = dP_c - dP_A \) and dividing this total derivative by \( dC \) and evaluating it at \( dP = 0 \) yields \( V - W \). The functional forms that are commonly used for the net pollution function \( P \) imply \( V - W < 0 \) (Lieb 2002, p.434). Thus the right-hand side of expression (3) describes the negative of the change of the slope of the consumption possibilities curve (that is, the change in \( MRT^C.P \)) as the consumer moves to higher and higher consumption possibilities curves

\(^3\) Lieb does not use superscripts in his notation for the marginal rate of substitution. We added them to distinguish Lieb’s marginal rate of substitution between consumption and pollution from C&B’s marginal rate of substitution between consumption and environmental quality.

\(^4\) The total derivative of the net pollution function \( P(C, A) \) is \( dP = P_c dC + P_A dA \). Because \( dA = dY - dC \), evaluating this expression at \( dY = 0 \) implies \( dP = P_c dC - P_A dC \) and yields \( MRT^C.P = dP/dC|_{dY=0} = P_c - P_A \).
(dY > 0) while keeping the level of pollution unchanged (dP = 0 and therefore dC > 0).5

Expression (3) therefore states simply that net pollution decreases with income

(dP/dY < 0) if $MRS^{C,P}$ falls more than $MRT^{C,P}$ as income increases—that is, if the indifference curves become flatter relative to the consumption possibilities curves, evaluated at any given level of pollution—and vice versa.

2) C&B:

For their comparative static analysis, C&B normalize $N = 1$ so that there is no difference between per capita consumption $c$ and aggregate consumption, which we denote as $C$. Their Proposition 1 states that the direction of change in environmental quality in response to a change in output is determined by

$$\frac{dE}{dA} > 0 \Leftrightarrow \frac{\Pi_{xx} d\pi}{\Pi_x} > \left(\frac{U_{CC}}{U_C} - \frac{U_{EC}}{U_E}\right) \frac{dC}{dA}. \quad (4)$$

We now establish equivalence of expression (4) and expression (3). C&B use the utility function $U = U(C, E)$ so that $MRS^{C,E} = -\frac{U_C}{U_E}$. This implies

$$MRS^{C,E}_C = \frac{\partial MRS^{C,E}}{\partial C} = \left(\frac{U_{CC}}{U_E} - \frac{U_C U_{EC}}{(U_E)^2}\right) = -\frac{U_C}{U_E} \frac{U_{CC}}{U_C} \frac{U_{EC}}{U_E}, \quad (5)$$

and using (5) in expression (4) yields

$$\frac{dE}{dA} > 0 \Leftrightarrow \frac{U_C}{U_E} \frac{\Pi_{xx} d\pi}{\Pi_x} > MRS^{C,E}_C \frac{dC}{dA}. \quad (6)$$

5 Lieb (2002, p.435) illustrates this in a diagram with $C$ and $P$ on the horizontal and vertical axis, respectively, with monotonically increasing convex consumption possibilities curves and monotonically increasing concave indifference curves.
Optimal provision of environmental quality (a public good) requires that the Samuelson condition be satisfied in steady state: the discounted marginal rate of substitution between environmental quality and consumption, summed over all individuals, must equal the marginal rate of transformation between environmental quality and consumption. C&B’s model assumptions imply that in steady state an increase in consumption is exactly offset by a decrease in environmental protection effort. Because a decrease in environmental effort reduces environmental quality by $\pi / \Pi$ at the margin, the marginal rate of transformation between environmental quality and consumption is

$$MRT^{E,C} = \frac{1}{\Pi \pi}.$$  

Thus in C&B’s model the Samuelson condition is

$$N \frac{1}{\rho} \frac{U_E}{U_C} = \frac{1}{\Pi \pi}$$

is C&B’s equation (4), p. 370).

When $N = 1$, the Samuelson condition implies that

$$\frac{U_C}{U_E} = \frac{\Pi \pi}{\rho}.$$ 

Substituting this in expression (6), and dividing both sides of the expression by $\frac{dC}{dA} > 0$ yields

$$\frac{dE}{dA} < 0 \Leftrightarrow \frac{\Pi \pi}{\rho} \frac{d\pi}{dC} \leq \frac{MRS^{C,E}}{\Pi \pi}.$$  

(7)

Because $\Pi$ is a function of $\pi$ only, its total derivative is $d\Pi_{\pi} = \Pi_{\pi \pi} d\pi$ so that

$$\frac{d\Pi_{\pi}}{dC} = \Pi_{\pi \pi} \frac{d\pi}{dC},$$

which permits us to rewrite expression (7) as

$$\frac{dE}{dA} > 0 \Leftrightarrow -\frac{MRS^{C,E}}{\rho} \leq \frac{1}{\Pi \pi} \frac{d\Pi_{\pi}}{dC}.$$  

(8)

The next step is to evaluate expression (8) in terms of abatement $\Pi$ and the stock of net pollution $P^N$, instead of environmental quality $E$. The steady state stock of
environmental quality $E$ is the difference between the steady state values of gross pollution, $P(K)$, and environmental protection, $\Pi(\pi)$, so that $P^N$ equals

$$P^N = -E = P(K) - \Pi(\pi).$$  \hspace{1cm} (9)

Taking the derivative of $P^N$ with respect to $\pi$ yields

$$P^N_\pi = P_k \frac{\partial K}{\partial \pi} - \Pi_\pi.$$

That is, the change in steady state net pollution due to an increase in environmental effort is equal to the difference between the change in gross pollution that results from reallocating resources from the capital stock to environmental effort and the increase in pollution abatement.

The equation of motion for capital allows us to determine the steady state stock of capital as

$$K = F(K) - C - \pi.$$  \hspace{1cm} (11)

Equation (11) implies that the capital stock and environmental effort in the steady state are directly inversely proportional to each other, so that $\frac{\partial K}{\partial \pi} = -1$ and therefore

$$P^N_\pi = -P_k - \Pi_\pi,$$

so that

$$\frac{dP^N_\pi}{dC} = -\frac{dP_k}{dC} - \frac{d\Pi_\pi}{dC}.$$  \hspace{1cm} (12)

Taking the derivative of $P^N$ in equation (9) with respect to consumption and using the fact that $\frac{\partial K}{\partial C} = -1$ yields
\[ P_C^N = P_k \frac{\partial K}{\partial C} = -P_k \]  

(13)

That is, the one-to-one trade-off between consumption and the capital stock in steady state implies that the increase in net pollution that results from an increase in steady state consumption equals the decrease in gross pollution caused by the decrease in the steady state capital stock. Furthermore, equation (13) implies that the change in the marginal impact of consumption on net pollution that results from additional consumption is exactly offset by the change in the impact of capital stock on gross pollution, or

\[ \frac{dP_C^N}{dC} = -\frac{dP_k}{dC} . \]  

(14)

Thus substituting (14) into (12) and rearranging terms yields

\[ \frac{d\Pi}{dC} = \frac{dP_C^N}{dC} - \frac{dP_N^N}{dC} , \]  

(15)

The left hand side of equation (15) is the change in the marginal effectiveness of environmental protection effort due to an increase in consumption, which is also the change in the marginal rate of transformation between consumption and net pollution due to an increase in consumption.

Using (15) in (8) leads to

\[ \frac{dE}{dA} = 0 \iff -MRS_{C,E}^C = \frac{1}{\rho} \left( \frac{dP_C^N}{dC} - \frac{dP_N^N}{dC} \right) . \]  

(16)

Equation (9) implies \( \frac{dE}{dA} = -\frac{dP_N^N}{dA} \), so that expression (16) becomes
The assumption that environmental quality $E$ is the difference between some pristine environmental quality, normalized to zero, and net anthropogenic pollution implies $U_p = -U_e$, so that $MRS_{c,e} = -MRS_{c,p}$ and $MRS_{c,e} = -MRS_{c,p}$, and therefore

$$\frac{dp^N}{dA} > 0 \iff MRS_{c,p} < \frac{1}{\rho} \left( \frac{dp_N}{dC} - \frac{dp^N}{dC} \right)$$

(17)

or

$$\frac{dp^N}{dA} > 0 \iff -MRS_{c,p} \geq \frac{1}{\rho} \left( \frac{dp_N}{dC} - \frac{dp^N}{dC} \right)$$

(18)

(19)

The remaining differences are notational. First, for C&B, differences in total factor productivity $A$ are the only source of differences in effective resources across countries in the steady state, so that $dA$ is the same as $dY$ in Lieb’s notation. Second, in comparing C&B’s dynamic model with Lieb’s static model, note that C&B’s model is evaluated at the steady state at which all variables, including net pollution, are constant, so that $\frac{dp_N}{dC}$ and $\frac{dp^N}{dC}$ are evaluated at $dp^N = 0$. The variables $\pi$ in C&B and $A$ in Lieb both represent the use of resources for pollution abatement, so that $\frac{dp^N}{dC}$ in C&B’s notation is the same as $\frac{dp_A}{dC}$ in Lieb’s notation. Similarly, $\frac{dp^N}{dC}$ in C&B’s notation is the same as $\frac{dp_C}{dC}$ in Lieb notation. Finally, the discount rate equals 1 in a static model by construction, so that expression (19) is identical to expression (3).
4. Interpretation and conclusion

Expression (3) is a very general condition for an EKC in the types of models developed by C&B and Lieb. It illustrates that only appropriate joint changes of MRS and MRT may yield an EKC. By itself, expression (3) does not imply any particular driving force behind the EKC. This suggests that C&B’s result is not driven by differences in total factor productivity, per se, but rather by differences in income/resources, regardless of what the source of those differences might be.

C&B find that decreasing returns to scale in environmental protection yield a cross-sectional EKC across countries because they ensure that additional environmental expenditures increase the cost of improving environmental quality by more when environmental effort is low than when it is high (C&B, p.372). However, the equivalence between C&B and Lieb’s results suggests that such decreasing returns to scale simply ensure that $-\frac{\text{MRS}}{\rho} > -\frac{1}{\rho} \cdot \frac{dMRT}{dC} \bigg|_{\rho=0}$ when total factor productivity is small but that the inequality reverses at higher levels of total factor productivity. Although C&B’s observation is certainly correct, this condition can be fulfilled through alternative restrictions on either preferences or the pollution function as well.

Virtually every paper on the EKC written in the past decade has been careful to emphasize that economic growth and technological progress alone do not guarantee that pollution will ultimately fall with income. C&B also emphasize in their conclusion that A comparison of the transitional dynamics of the model with our comparative statics results reveals that economic growth does not necessarily imply eventual environmental improvements, even if the cross-country empirical evidence suggests otherwise. Thus, socio-economic reforms that promote macroeconomic
efficiency primarily through the removal of barriers to technology adoption will not necessarily improve environmental quality.

Disclaimers of this sort highlight what a paper does not show. We are not aware of any empirical analysis or theoretical model of the EKC, which suggests that economic growth by itself can lead to an EKC regardless of people’s preferences for environmental quality. In fact, Plassmann and Khanna (2006) show that under very general assumptions about consumer preferences and technology, economic growth and technological advances alone can never ensure that pollution will ultimately decline as income increases. Economic growth enables consumers to increase their expenditures on abatement (or environmental protection) without having to reduce consumption. Technological progress in abatement technologies lowers the relative price of improving environmental quality. Whether these incentives are sufficiently strong for consumers to adjust their behavior in a way that leads to higher environmental quality depends entirely on the consumers’ preference structures. By establishing the equivalence between C&B’s and Lieb’s results, our analysis underscores this point.

\textbf{Acknowledgements:} We thank Jon Conrad for useful discussions and two anonymous referees for constructive comments on a previous draft. All errors are ours.

\footnote{Plassmann and Khanna’s (2006) result does not require assumptions about the curvatures of the pollution and abatement functions; it is therefore more general than Lieb’s and C&B’s result that is derived under specific assumptions about the signs of the second order derivatives of these functions.}
References


