A Dynamic Regional Applied General Equilibrium Model

with Five Factors of Production

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Abstract

Since the mid 1980s, applied general equilibrium (AGE) models have been used to analyze the effects of regional economic policies. Unfortunately, most regional AGE models use assumptions that are too restrictive to yield reliable results. This paper presents an AGE model for regional policy analysis that has four major improvements compared to existing regional AGE models. First, the model is dynamic, which permits a more coherent description of intertemporal optimization decisions than is possible in a static model. Second, the model incorporates both the sale value and the rental value of assets, which permits the inclusion of investment projects whose returns consist of mixtures of payments and capital gains or losses. Third, the model uses five factors of production: land, labor, buildings, machines, and infrastructure, which differ in mobility and reproducibility. Fourth, the model replaces the generally used assumption that goods from a given industry that are produced in different regions are imperfect substitutes (the Armington assumption) with the assumption that these goods are perfect substitutes, which is more realistic for regional models given the data that are commonly used for the analysis.

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1. Introduction

The falling cost of computing power during the last twenty years has made applied general equilibrium (AGE) models a widely used tool for the analysis of economic policy. Most AGE models are extensions of analytically solvable two-sector general equilibrium models. A general equilibrium model describes the circular flow of input factors and output products between consumers and producers, and, as opposed to a partial equilibrium model, emphasizes that economic actions may have impacts on relative prices throughout the economy, now and in the future. A solution to a general equilibrium model consists of a set of prices for all goods that ensures that, in all markets, quantity supplied equals quantity demanded. Only under very restrictive model assumptions it is possible to find this set of prices with analytic methods. Models with numerous sectors, agents, and regions are usually too complex to have analytic solutions, and they need to be solved numerically.¹

AGE models have sometimes been used for the analysis of policies that affect a single region with little or no connection to other regions, as well as for the analysis of policies that affect various regions or countries simultaneously. These models differ in the degree to which they attempt to be realistic. Models that are used to examine the effects of progressive income tax changes require agents that belong to different income groups, while for trade models that describe the effect of tariffs on consumption, the number of different agents is less important than the number of sectors and the degree to which imported goods are substitutes for domestic goods. Some models assume that output is produced solely from the agents’ original factor endowments, while other models assume that production also requires the input of intermediate goods. In many cases, the availability of data restricts the level of detail that can be achieved.

Since the mid 1980s, regional AGE models have become an increasingly popular tool for economic analysis, because they permit the analysis of regional policies and because they reduce the aggregation problems of analyses on the national level. Many regional models use assumptions similar to those used by models on the national level, and interactions among regions are frequently modeled very similarly to the interactions among countries in the trade AGE literature. Unfortunately, these practices introduce unnecessary biases into regional AGE models.

First, while most AGE models on the national level are multi-period dynamic models, almost all existing regional AGE models are static one-period models. This is not too surprising, because most national models avoid many of the difficulties that emerge from the existence of several closely interrelated regions, for example the question of how to describe migration across regions, changes in jobs across regions and industries, investment in several region-specific assets, the degree of substitutability of goods that are

¹ AGE models are generally developed within the neoclassical framework. On the consumer side, agents are assumed to be rational and utility maximizing; they are endowed with sets of factors that they exchange, either among each other or with producers, to maximize their utility. Producers transform agents’ factor endowments into output, which they sell to obtain funds to buy factors from agents. Markets are generally assumed to be competitive, so that producers do not make positive profits after paying all factors their market returns, and very often production is assumed to take place under constant returns to scale.
produced in different closely interrelated regions, and so on, which can be difficult to describe in a dynamic model in which agents have perfect foresight over several periods. However, economists believe that consumption and savings decisions are understood best in a dynamic framework, and a static model is unlikely to describe these decisions adequately.

Second, we are not aware of even dynamic AGE models on the national level that incorporate selling prices of assets. Use of this information permits returns to investments to be combinations of payments and capital gains. Since the possibility of capital gains is an important factor in many investment decisions, for example in the market for housing, a model that ignores the possibility of capital gains and losses will not describe investment decisions accurately.

Third, models on the national level generally combine factors of production into only two factors, labor and capital; capital is frequently assumed to be internationally mobile, while labor is usually assumed to be mobile only among industry sectors. Regional models often ignore immobile factors like infrastructure and land, and make overly simplified and unrealistic assumptions regarding the mobility and availability of labor and capital. These models are therefore likely to misrepresent the effects of regional policies on factor availability.

Finally, in the trade AGE literature, goods that are traded among countries are assumed to be imperfect substitutes for each other. This helps to model the existence of interindustry trade across countries, and implies that every agent in every country demands all goods that are produced in all countries. In regional models, the assumption that a consumer is willing to pay different prices for otherwise identical goods from all regions is rather unrealistic given the data that are used to describe an economy, yielding a distorted representation of the consumer demand structure.

We therefore suggest that regional AGE models that are used for policy analysis should possess the following four properties. They should

1. be dynamic rather than static models,
2. incorporate the possibility of capital gains and losses in the return to investment projects,
3. incorporate at least five different factors of production (land, labor, immobile capital, mobile capital, and infrastructure) with their respective properties of durability, reproducibility, and mobility, and
4. treat consumption goods that are produced in different regions as perfect substitutes.

In the following section we offer reasons of why we think that these properties are essential properties for regional AGE models. Section 3 reviews the regional AGE literature, noting which existing models possess which of these properties. In Section 4 we describe a regional AGE model that possesses the four properties we seek, and Section 5 reports the results of an experiment with our model using artificial data, which helps explain why we believe our model will yield improved estimates of the effects of regional and federal policies. Section 6 summarizes and concludes.
2. Desirable properties of a regional AGE model

2.1. The model should be dynamic.

A general equilibrium model is meant to describe the flows of factors and products in an economy. The decisions that households make about these flows depend on future as well as present opportunities (i.e. prices). A one-period model that assumes that households do not save and that no investment takes place (assumptions that are made frequently in regional AGE models), must also assume that all capital is supplied inelastically. Such a model ignores the fact that capital must be produced, and that all capital depreciates over time; an analysis of a policy that affects the return to capital will substantially underestimate the policy’s long term impact on the availability of capital and on the productivity of the region. An additional disadvantage of the absence of saving and investment from a model is that such a model will be unable to accommodate the existence of budget deficits or surpluses as well as trade deficits or surpluses, and therefore it will not account correctly for the uses of income.

In a one-period model, mobile factors necessarily respond to policy changes immediately. While this assumption might be justified in the case of highly mobile or rapidly depreciating machines (depending on the time horizon of the model), the response of labor is more likely to be gradual. Few persons are able to move to a different region or to change their job to one in a different industry immediately after a new policy has created an incentive to move. If labor is assumed to be completely mobile either across regions or industries, a static model will overestimate the short-run effect of such a policy. If, on the other hand, labor is assumed to be completely immobile across regions or industries, a static model will underestimate the short-run effect. While dynamic models are more difficult to implement, we believe that their ability to better account for the intertemporal impact of economic policies justifies the extra effort.

2.2. The model should differentiate between the sale value and the rental value of an asset

We are not aware of any AGE model that incorporates the selling prices (the present value of future returns) of factors. Determination of the selling prices of factors is important for three reasons: first, since a household’s intertemporal optimization decision is affected by the value of its initial assets, it is necessary to know this value to model the household’s decision correctly. A model that assumes that a household uses only information about the income that accrues in the present period when making its consumption and savings decision ignores the impact that accumulated wealth has on this decision. Changes in future prices affect the current value of a household’s assets, and if the effects of policies on future prices are ignored, their effect on present consumption decisions will be estimated incorrectly.

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2 This is important because a household with a high stock of wealth is likely to spend a larger proportion of its income flow on consumption than a less wealthy household with the same income.
Second, very often the return to investment in structures and machines is a combination of factor payments and capital gains or losses, and, for some investment projects, the short-run return is entirely capital gains. Capital gains and losses can only be described in a dynamic setting that incorporates future prices of assets. If the selling price of a new investment project is unknown, one cannot determine capital gains or losses that are part of the return, and the model will be unable to describe investment decisions correctly.

Third, in equilibrium the cost of investing in a new unit of capital should equal the selling price of the unit, discounted from the period in which the new capital becomes available. This implies that there is a specific relationship among the cost of producing new capital, the prices of the services of capital in all future periods, the rate of depreciation, and the rate of return (i.e. the interest rate). A model that ignores this relationship is likely to suffer from inconsistencies.

2.3. *A regional AGE model should differentiate among at least five factors of production.*

Factors of production may differ from one another in at least three aspects: durability, reproducibility, and mobility across regions and industries; generally they are also imperfect substitutes for one another. Since economic policy affects the availability of factors of production in different ways, it is essential to model their properties explicitly and to aggregate them only to the extent that their properties are similar. We suggest that regional general equilibrium models should differentiate among at least five factors of production: infrastructure, labor, land, structures, and machines.\(^3\)

Infrastructure consists of publicly provided goods that enhance both consumption and productivity, as well as publicly provided goods that enhance only consumption. The first category includes highways, airports, harbors, fire and police protection, power lines, sewers, etc., while the second includes public parks, public libraries, schools, clean streets, etc. Since mobile factors of production move into and out of regions in response to changes in the provision of infrastructure, failing to incorporate infrastructure in a regional AGE model will lead to inadequate modeling of the incentives for mobile factors to move across regions as well as to an overestimation of the welfare cost of taxes on labor and capital that are levied to provide infrastructure.

Labor is moderately mobile, and is the only factor of production that is not fixed in supply at any given point in time. Many regional AGE models ignore the labor/leisure decision and assume that labor is supplied inelastically. This is almost never warranted, and leads to an underestimation of the effects of policies such as income taxes and sales taxes that create disincentives to work.

Often economic models treat labor as highly mobile, but this treatment ignores the fact that workers need time to relocate to different regions, and probably even more time to acquire new skills that permit them to

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\(^3\) Depending on the purpose for which the model is built, even more factors might be desirable. It would be possible to subdivide any of these factors to account for additional differences, but restrictions posed by the availability of data may make such further subdivisions problematic.
work in different industries, once economic policies have provided an incentive to do so. In a model that contains more than one industry or region, an assumption that labor is perfectly mobile across industries and/or regions will overestimate the effects of policies that affect migration incentives.

Many models combine infrastructure, land, structures, and machines into a single factor, ‘capital.’ But the only property that these factors have in common is that they are ‘not labor,’ while their individual properties and responses to economic policy differ greatly from one another.

Land is immobile, and generally very durable and non-reproducible. Although the supply of land is inelastic, it can be used with varying intensity. Economists generally agree that a tax on the value of land is a neutral tax that does not lead to a welfare loss, because it does not change the user’s differential returns to alternative uses of land.

Structures (immobile capital) are almost as immobile as land, but they are reproducible and depreciate over time. In the short run a tax on structures will not affect the availability of structures in a region, while a policy that reduces the incentive to erect new structures will lead to a reduction in the number of available structures in the region over a longer span of time.

Machines (mobile capital) are very mobile, reproducible, and depreciate faster than structures, partly because they become obsolete faster. A regional policy that reduces the incentive to use machines will lead to a rapid reduction in the number of available machines, because they will move to other regions. If a policy reduces the overall return to machines in all regions, it will lead, somewhat more slowly, to a decrease in the total number of machines by discouraging their production.

A model that combines land and structures into a single inelastically supplied factor ignores the fact that structures need to be constructed, and it will therefore underestimate the negative long-run effect of taxing this hybrid factor. If, on the other hand, structures and machines are combined into a single mobile factor, the model will ignore the immobility of structures and will therefore overestimate the negative short-run effect of taxing mobile capital.4

2.4. Consumption goods that are produced in different regions should be perfect substitutes

Models that describe international flows of goods and factors usually employ the assumption that goods produced in different countries are imperfect substitutes. This so-called ‘Armington assumption’ (Armington, 1969) of imperfect substitutability is used to explain the fact that very often two countries exchange goods that belong in the same category (‘cross-hauling’ of goods), and has the additional advantage that it greatly simplifies the model, because it permits (otherwise identical) goods that are produced in different countries

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4 For national-level one-region models that assume immobility of capital across countries, combining structures and machines into a single factor creates less of a problem, although the assumption of international capital immobility is somewhat questionable. However, assuming that capital is immobile or that capital owners are unable to relocate can lead to the erroneous conclusion in one-period models that a tax on capital is non-distortive (see, for example, Morgan, Mutti, and Rickman, 1996, p.138).
to have different prices. If goods from different countries were perfect substitutes and production conditions differed among countries, utility maximizing consumers would buy only from the country that was the cheapest producer, which would result in complete specialization. The Armington assumption helps avoid this counterfactual result, and permits countries in the model to share markets despite having different costs of production.

On first view the Armington assumption seems to be justified in a regional model as well. After all, food that is produced in Maine (e.g. lobsters) differs from food that is produced in Virginia (e.g. ham), and for most consumers the two are imperfect substitutes. Unfortunately the data that are commonly used in AGE models do not support this view. AGE models aggregate all produced goods into output from a relatively small number of industries (depending on the size of the model, the number of industries varies from 5 to about 30); generally, lobsters and ham are both assumed to be produced by the food industry. Since Maine and Virginia produce more food products than only lobsters and ham, the production function of the food industry in each state must describe the combined production of all food products. Estimates of the parameters of the production function of ‘food’ are available on the federal level, but usually not on the state level; the best assumption about production is therefore that industries that produce the same group of goods have identical production functions across states.

In addition, available data about consumer demand indicate the region- and consumer-specific demand for a particular good, but do not provide information about demand for region-specific products. Although the consumer expenditure survey by the Department of Labor Statistics divides demand for consumption goods into various sub-categories, these subcategories generally do not provide information about the region of origin of the goods. For example, the survey states the amount consumers spend on fresh vegetables, but does not indicate how much is spent on potatoes from Idaho and how much on beans from Texas, and although the share that consumers spend on fish and seafood is known, no information is available how much fish and seafood is purchased from states at the Atlantic and from states at the Pacific, and how much fish was caught in rivers and lakes. For non-food categories, identifications of this sort are even more difficult. Given these data restrictions, it is more reasonable to assume that goods produced in different regions that are otherwise identical are perfect substitutes, so that the equilibrium prices of all such goods that are consumed in a given region will be equal in that region.\(^5\)

Besides its intuitive appeal, an additional advantage of this assumption is that it makes it unnecessary to estimate the elasticities of substitution for identical goods from different regions (for example, the elasticity of substitution between all food that is produced in Maine and all food that is produced in Virginia), which

\(^5\) One might argue that prices should differ among regions because the transportation of goods between regions is costly. To meet this criticism, most AGE models assume that output is produced with a combination of intermediate goods and additional factors of production, where one of the intermediate goods is ‘transportation.’ Buckley (1992) and Bröcker (1998b) have developed frameworks for static regional AGE models that incorporate transportation cost, but still use the Armington assumption.
generally enhances the precision of the model. We are not aware of any AGE model that employs the assumption of perfect substitutability of identical goods across regions. The most likely reason is that this assumption makes the model considerably more complicated. Under the assumption that production functions for identical goods are identical for all regions, it is not at all straightforward to maintain identical output prices in all regions while permitting input prices to vary across regions. However, we consider the extra computational effort to be worthwhile because it provides internal consistency and greater precision.

Existing regional AGE models do not incorporate most of these properties that we regard as essential, and policy simulations undertaken with those models are therefore at best imprecise. The next section provides a brief survey of the existing regional AGE literature to motivate the use of our more detailed model.

3. The regional AGE literature

Although there are methods of drawing conclusions about regional effects from national models (‘top-down models’), most regional models explicitly incorporate the existence of more than one region (‘bottom-up models’). Table 1 provides a comparative summary of the properties of these multi-regional models. For each model the table shows the number of regions and the number of industry sectors to which the model is applied, its intertemporal structure, whether the model incorporates labor, capital, land, and public goods, and the mobility and supply assumptions made with respect to each of these factors. In addition, the table indicates whether a model assumes that output is produced with intermediate goods (using an input-output matrix), whether households are assumed to consume their total income or to divide their income between consumption and saving, whether governments are assumed to maintain balanced budgets (the existence of budget deficits requires household saving, so models without saving necessarily have to assume that all budgets are balanced, or at least that budget deficits remain constant), whether the model differentiates between traded and non-traded output goods, whether the model explicitly incorporates transportation cost, whether the model explicitly incorporates money, and the main purpose for which the model was developed.

All of the models that assume interregional mobility of goods use the Armington assumption that otherwise identical goods that are produced in different regions are imperfect substitutes for one another. With the notable exception of Feltenstein (1997), all models are static one-period models, and with the exceptions of Hoffmann et al. (1996) and Feltenstein (1997), all models assume that governments have either balanced budgets or exogenously determined budget deficits. None of the models incorporate the selling

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6 Broadway and Treddenick (1978) developed a general equilibrium model without the Armington assumption by assuming that products are homogenous among countries.

7 See, for example, Higgs and Powell (1990), who use Australia’s national-level ORANI model for regional analyses.

8 Kimbell and Harrison (1984) make the assumption that regions do not trade output products with each other, so the question of the interregional substitutability of products does not arise in their model.
prices of factors, which means that all models ignore both the welfare effects and the incentive effects of capital gains and losses on savings/consumption decisions.

Most models combine land, structures, and machines into a single factor ‘capital;’ Kraybill et al. (1992) and Buckley (1992) model a land/structure hybrid by assuming that capital is interregionally and intersectorally immobile, while all other models assume that capital consists of interregionally and intersectorally mobile machines only. Immobile ‘land’ is introduced explicitly only in Kimbell and Harrison (1984), Sehili (1997), in the models by John Whalley (Whalley and Trela, 1986; Jones and Whalley, 1988, 1989), and in the models by William Morgan and John Mutti. Public goods have an impact on the location decisions of mobile factors only in the models by Sehili (1997) and Hirte (1998); in the other models, public goods are used only to dispose of tax revenue without having any effect on factor productivity and availability. Many of the models assume that labor is supplied perfectly inelastically, and almost all models assume this of capital supply as well. Few of the models in Table 1 assume that households can determine endogenously how much of their income they want to consume; the others do not incorporate a savings decision at all.

Because of these restrictive assumptions, few of these models are able to describe the regional effects of economic policies accurately. For example, Jones and Whalley (1988, 1989) use their model to analyze the effects of changes in federal and regional taxes in Canada in the early 1980s. They estimate that the regional effects of tax changes, measured in terms of Hicksian equivalent variations, are stronger than the overall net effect, which is not surprising, because in their model labor is assumed to be supplied inelastically and to be mobile interregionally but not internationally. They find that most policy changes have small regional welfare effects; only the replacement of all federal and regional taxes and transfers by a uniform, yield-preserving sales tax shows large gains for some regions, while inducing large losses for others. The overall gain for Canada from this policy change is estimated to be 2.17 percent of GDP. This low estimate of the distortive effect of taxes on labor and capital is almost certainly due to the assumptions that labor and capital are supplied inelastically, that households do not save, and that tax effects on future prices do not affect the current consumption decision of households.

The model by William Morgan and John Mutti is used to examine the incidence and welfare effects of various tax changes in the U.S. in the early 1980s. Mutti, Morgan, and Partridge (1989) examine the incidence of a one percent reduction in the ‘business tax’ rate (presumably the corporate income tax) together with either a tax on land or increases in individual income taxes to keep revenue unchanged. Capital owners

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9 Morgan, Mutti and Partridge (1989) describe capital as ‘capital improvements,’ which seems to indicate that capital is assumed to include structures (which are commonly called ‘improvements’), but this is ruled out by their assumption that capital is perfectly mobile.


11 In their Table 3, Footnote b, they claim that this will leave national welfare unchanged.
are shown to gain from both policy changes, although a model that incorporates an endogenous savings decision is likely to show an even higher gain. Land owners are shown to bear the burden of the increase in the tax on land, and labor is shown to bear the net burden of the individual income tax. However, because a decrease in taxes on labor and capital should lead to an increase in the supply of labor and capital and thereby to an increase in the rent of land, a model that incorporates investment decisions and elastically supplied labor is likely to show a lower burden for land owners. Similarly, because a decrease in taxes on capital will lead to a higher supply of capital and thereby to a higher productivity of labor, the actual burden of labor is likely to be smaller.

One of the policies examined in Morgan, Mutti, and Partridge (1989) is the replacement of all federal and regional taxes by a federal non-distortive lump-sum tax; the excess burden of present U.S. taxes is estimated to be 0.6 percent of revenue. Although they note that the lump-sum tax could be imposed on land, the actual lump-sum tax used in their empirical model seems to be a head tax, which is non-distortive only because labor is assumed to be immobile internationally, and never so poor as to be unable to afford the tax. Morgan, Mutti, and Rickman (1996) undertake the same experiment with a similar result, but they impose the ‘lump-sum tax’ on capital income. In their model a tax on capital income is non-distortive only because capital owners are immobile, capital is immobile internationally, and because by assuming that households do not save, the model ignores the fact that capital is not actually supplied inelastically, but depreciates and must be replaced if it is to be used in future periods. Their estimate of the welfare cost of federal and regional taxes is therefore too low to be believable.

The other one-period models have similar limitations. To our knowledge, Feltenstein’s (1997) model is the only dynamic regional AGE model so far. His model describes the supply of labor and capital as the outcome of an intertemporal utility-maximization process, and the predictions of his model are therefore more believable than the those of models that assume that capital and labor are supplied inelastically. However, the model describes only two regions, employs the Armington assumption, assumes that households make their optimization decisions by disregarding their initial wealth, and imposes the constraint that households cannot borrow against their future income. In addition, because the model assumes that capital is interregionally immobile, it is unable to describe the effects that taxes on capital might have on interregional capital flows.

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13 It is well known in the local public finance literature that a head tax can be implemented by local communities to distort people’s migration decisions (Hoyt, 1991; Henderson, 1994).

14 They assume that income from the use of land and capital accrues to residents as well as non-residents, but unlike workers, capital owners are assumed to be unable to migrate to more attractive regions.
4. A regional AGE model with all four desired properties

We have developed a regional AGE model that possesses all four desired properties. The framework is flexible enough to accommodate any number of regions, sectors, agents, and time periods; the only limitations are posed by the availability of data used to calibrate the model and by the computing power that is available. This section describes the main properties of our model.

4.1. Factors of Production

The model incorporates five factors of production: land, structures, infrastructure, machines, and labor. Land, structures, and infrastructure are assumed to be interregionally immobile but intersectorally mobile. Land is supplied inelastically across all periods, while structures and infrastructure depreciate over time. The supply of structures in a region can be increased through private investment, while infrastructure is assumed to be a pure public good whose supply is determined by the amount of government investment. The mobility assumptions imply that the price for the use of land can vary across regions, as can the price for the use of structures, if structures are not being built in some regions. The price for the use of infrastructure is zero; infrastructure is financed solely through taxation.

Machines are assumed to be interregionally and intersectorally mobile; they depreciate over time, and new machines can be created through private investment. Because machines can be used in all regions and sectors and are able to move to the region in which they can earn the highest return, the equilibrium price for the use of machines, net of taxes, is identical in all regions.

Within any given period, labor is assumed to be sector-specific and interregionally immobile. Between any two periods, households can relocate to different sectors and regions. We believe that this mobility assumption captures labor market responses to changing conditions better than either the assumption that households can relocate immediately in response to changes in incentives, or the assumption that households are completely immobile.

4.2. Production, the elasticity of substitution among identical goods produced in different regions, and output prices

Production in each sector requires the input of intermediate goods and the five factors described in 4.1., which are used to produce value added. Requirements of intermediate goods for each sector are determined
according to an input-output table, and value added is calculated using a CES production function. We assume that the production function for value added for Sector \( s \) in Region \( r \), \( VA_{sr} \), is multiplicatively separable in infrastructure and the other four factors, so that

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VA_{sr} = f_s(I_{sr})(a_{1s}T_{sr}^{\beta_s} + a_{2s}S_{sr}^{\beta_s} + a_{3s}M_{sr}^{\beta_s} + a_{4s}L_{sr}^{\beta_s})^{\frac{1}{\beta_s}},
\]

where \( f_s(I_{sr}) \) describes the impact of infrastructure on value added, \( T_{sr} \) is the amount of Region \( r \)’s land (Terra) that is used in Sector \( s \), \( S_{sr} \) are structures, \( M_{sr} \) are machines, \( L_{sr} \) is labor, \( a_{is} \), \( i=1..4 \), are industry specific parameters related to the factor shares of their respective factors, and \( \beta_s = (\sigma_s - 1)/\sigma_s \), where \( \sigma_s \) is the elasticity of substitution in Sector \( s \) between any two factors from the set \{ \( T, S, M, L \) \}. Since technology is mobile within a national economy, the assumption is made that every sector’s production function is identical for all regions. All markets are assumed to be perfectly competitive so that firms do not make positive profits, which means that output prices are equal to the cost of the intermediate input goods plus the factor costs to produce value added. 

Because of the assumption that firms operate within perfectly competitive markets, the number of prices that must be solved for can be reduced to the number of factors. This means that a solution to the model with \( r \) regions, \( s \) sectors, and \( t \) time periods consists of a set of \((r+r+rs+1+1)\cdot t\) prices: each of the \( t \) periods has \( r \) prices for immobile land, \( r \) prices for immobile structures, \( rs \) prices for temporarily immobile and temporarilily sector-specific labor, one price for mobile capital, and one extra ‘price’ to solve a simultaneity problem with respect to government revenue described in Section A.8. below. In each iteration, the solution algorithm chooses a set of prices that is used to determine excess demand (quantity demanded minus quantity supplied) for each factor. Once all excess demands are zero (or almost zero, depending on the degree of precision attempted), an equilibrium solution has been found.

If sector output prices for traded goods differ across regions, complete specialization can be prevented by making the Armington assumption that goods from different regions are imperfect substitutes for each other. However, as was mentioned in the main text in Section 2.4., it is difficult to motivate the use of the Armington assumption in a multi-region model of a nation. An alternative to the Armington assumption is the (more realistic) assumption that sector output prices are identical in all regions, so that households are indifferent among goods that are produced in the same sector in all regions. Since markets are assumed to be competitive, sector production cost must be identical in all regions. However, if the prices of all factors

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16 See Ballard, Fullerton, Shoven and Whalley (1985), Ch.3 for a detailed description of how to combine value added and intermediate goods.

17 It will be technically straightforward to use different parameters for each region if this is justified for a particular application of the model, and if the required data are available.

that are used in a sector were set independently across regions, equal production costs in this sector across regions would be obtained only by chance. To ensure that sector production cost are equal, the price of one factor per region and sector must be determined as a function of total sector production cost and all other factor prices in this region. This implies that in each region at least one factor must be sector-specific; we believe that labor is more likely to be sector-specific than either land, structures, or machines, but any of these factors could be made sector-specific if justified for a particular application of the model.

Prices of sector- and region-specific labor are determined as follows: instead of searching for \( rs \) prices for labor, the convergence algorithm searches for only \( s \) prices—the prices for labor in all sectors in Region 1, so that only \( (r+r+s+1+1) \cdot t \) prices are chosen for each iteration. The rental prices of land, structures, and labor in all sectors in Region 1, together with the rental price of machines, are used to determine the output prices for each sector in Region 1, which are then used as the sector-output prices in all regions. Given these output prices, the price of machines, and the region-specific prices of land and structures, the price of labor in each sector in each region is set to yield production cost for that sector in that region that is equal to the output price that was determined for that sector in Region 1.\(^{19}\) An equilibrium solution therefore consists of a vector of \( (r+r+s+1+1) \cdot t \) prices for which all factor markets clear simultaneously.

To ensure that the algorithm converges to an equilibrium in which labor markets in all regions and sectors clear simultaneously, the model uses the assumption that consumers are indifferent among goods that are produced in the same sector in all regions. After having determined labor supply in each sector and each region from the households’ utility maximizations, the total amount of output that can be produced in each region and each sector with this labor supply is calculated, and these numbers are summed to obtain the economy-wide output for each sector. After determining economy-wide demand for goods in each sector, the ratio of economy-wide demand to economy-wide supply, \( x \), can be calculated for each sector. For each sector, we assume that each region produces \( x \) percent of the maximum output capacity of the sector in that region to guarantee that all regions have either excess demand for or excess supply of labor. Once \( x=1 \) for a sector, the labor markets for this sector clear simultaneously in all regions.

4.3. The selling and rental prices of factors and their relationship to the interest rate

The selling price of a factor is the present discounted value of all future prices for the use of its services. Calculation of the present discounted value requires knowledge of the return that the factor yields in each period; for reproducible factors, this return depends on the cost of investment, that is, the cost of producing additional units of the factor.\(^{20}\)

\(^{19}\) The functional form of (1) does not permit an algebraic solution for the price of labor, so that the sector- and region-specific prices of labor need to be determined numerically.

\(^{20}\) This poses a difficulty for AGE models: while in reality output is produced with various mobile factors like computers, trucks, tools etc, sufficient data are rarely available to permit the use of such disaggregated production functions. To overcome this
The return to a specific reproducible factor, for example machines, is determined as follows: given the price of services of machines in the present and all future periods, the period $t$ selling price of a machine that is available for immediate use, $P_t$, is

$$P_t = p_t + \frac{p_{t+1}(1 - \delta)}{(1 + r^M_t)} + \frac{p_{t+2}(1 - \delta)^2}{(1 + r^M_t)(1 + r^M_{t+1})} + \ldots, \tag{2}$$

where $p_t$ is the rental price of a machine in period $t$, $r^M_t$ is the rate of return to machines in period $t$, and $\delta$ is the rate at which the machine depreciates per period.\(^{21}\) Period $t$’s selling price of a machine that will be available for use in the subsequent period, $P_t^*$, is given by

$$P_t^* = \frac{P_{t+1}}{(1 + r^M_t)} + \frac{p_{t+2}(1 - \delta)}{(1 + r^M_t)(1 + r^M_{t+1})} + \ldots \tag{3}$$

From (2) and (3) follows that $P_t = p_t + (1 - \delta) P_t^*$: period $t$’s selling price of a machine is equal to period $t$’s rental price of the machine plus period $t+1$’s selling price of that part of the machine that did not depreciate. This implies that whenever new investment in machines takes place, $P_t^*$ must be equal to the investment cost of building one new machine in period $t$—if the investment cost was lower than $P_t^*$, nobody would want to buy an existing machine, and if the investment cost was higher than $P_t^*$, nobody would want to invest in a new machine, but everybody would want to buy existing machines.

Similarly, period $t+1$’s selling price of a machine that will be available for use in $t+1$, $P_{t+1}$, is given by

$$P_{t+1} = p_{t+1} + \frac{p_{t+2}(1 - \delta)}{(1 + r^M_{t+1})} + \frac{p_{t+3}(1 - \delta)^2}{(1 + r^M_{t+1})(1 + r^M_{t+2})} \ldots \tag{4}$$

and period $t+1$’s selling price of a machine that will be available for use in $t+2$, $P_{t+1}^*$, is equal to

$$P_{t+1}^* = \frac{P_{t+2}}{(1 + r^M_{t+1})} + \frac{p_{t+3}(1 - \delta)}{(1 + r^M_{t+1})(1 + r^M_{t+2})} + \ldots, \tag{5}$$

which is equal to the cost in $t+1$ of investing in one new unit of machinery. Using (3) and (4), $P_{t+1}$ can be expressed as

---

\(^{21}\) To simplify we assume that $\delta$ is constant over time. $\delta$ includes physical depreciation as well as obsolescence.
Note that (a) selling prices as well as the rate of return can be computed without having to estimate additional model parameters, and that (b) the rental prices of factors, their rates of depreciation, and the cost of producing new reproducible factors determine a specific rate of return to investment in this factor; neglecting this relationship, for example by treating the return to investment as an additional unknown of the model, is likely to result in internally inconsistent models.

Feltenstein and Morris (1990, pp. 338-339) assume that different assets are imperfect substitutes for each other, and that households choose to invest in assets according to their relative yields.

\[ P_{t+1} = P_t^* (1 + r_t^M), \]  

(6)

and using (4) and (5) as

\[ P_{t+1} = p_{t+1} + (1 - \delta)P_t^*. \]  

(7)

Since the prices of services and the production costs of new machines in periods \( t \) and \( t+1 \) are known, \( P_{t+1} \) can be calculated from (7), and the rate of return to machines in period \( t \) can be determined from (6).\(^{22}\) Note that this rate of return includes factor payments as well as capital gains and losses, because \( r_t^M \) includes the change in the sale value of machines from period \( t \) to period \( t+1 \). Thus \( r_t^M \) is the return to new machines as well as to old machines, and investors are indifferent between buying an old machine and investing in a new machine, because both yield identical returns.

The return to a reproducible factor \( i \) at time \( t \), \( r_t^i \), is determined as a function of the rental prices for this factor in all future periods until the factor has depreciated completely, and the construction cost of a new unit of this factor, which implies that two different reproducible factors, for example machines (\( M \)) and structures (\( S \)), that have different prices of their services and/or different construction costs will most likely have different rates of return, \( r_t^M \) and \( r_t^S \), respectively. Since the model employs the assumption of perfect foresight for at least the following two periods, utility maximizing households will invest only in the factor with the highest return, and nothing in factors with lower returns. Avoiding this counterfactual result requires an ad hoc assumption that households are willing to invest part of their savings in factors with lower returns, which is difficult to motivate under perfect foresight.\(^{23}\)

A more coherent explanation of household investment in different assets is obtained by assuming that all factors in which investments will be made have identical returns. This can be achieved by determining the return to one factor as described above, and then adjusting the returns to all other factors by appropriate capital gains or losses until all factors yield identical returns, so that households are indifferent among investments in all factors.

If all factors yield identical returns, the framework of perfect foresight does not suggest an answer to the question of how much each household will invest in each reproducible factor. Investing everything in one factor is as consistent as dividing all savings equally among all factors. To approximate decisions made in a world with imperfect foresight by a model of perfect foresight, it is possible to use the fact that, even if all factors have identical returns, they still differ with respect to the composition of their returns. Identical

\(^{22}\) Note that (a) selling prices as well as the rate of return can be computed without having to estimate additional model parameters, and that (b) the rental prices of factors, their rates of depreciation, and the cost of producing new reproducible factors determine a specific rate of return to investment in this factor; neglecting this relationship, for example by treating the return to investment as an additional unknown of the model, is likely to result in internally inconsistent models.

\(^{23}\) Feltenstein and Morris (1990, pp. 338-339) assume that different assets are imperfect substitutes for each other, and that households choose to invest in assets according to their relative yields.
returns are achieved by assuming that factors with low future service prices carry a high capital gain (small capital loss), and factors with high future service prices carry a small capital gain (high capital loss). A high capital gain can be interpreted as expectation of large increases in future service prices (future = beyond the perfect foresight time horizon of the model), and a low capital gain as expectation of small increases in future service prices. An investment in factors with low service prices in the near future and high capital gains is therefore riskier than an investment in factors with high service prices in the near future and low capital gains. Under the assumption that households are somewhat risk averse, the share of investment in factor $i$ can be determined according to the ratio of the service price of factor $i$ in period $t+1$ to the cost of producing the factor in period $t$, $p_{\text{INV}_i}$, adjusted by the rate of depreciation, and weighted by the total value of the existing stock of the factor, $P_i x_i$, or

\[
\text{share}_i = \frac{\left(\frac{p_{\text{INV}_{t+1}}}{p_{\text{INV}_i}} - \delta_i \right) \cdot P_i x_i}{\sum_j \left(\frac{p_{\text{INV}_{t+1}}}{p_{\text{INV}_j}} - \delta_j \right) \cdot P_j x_j}^\alpha, \tag{8}
\]

where $\alpha$ is a parameter of the model that determines the degree to which returns other than capital gains determine the investment share in the factor; if $\alpha = 0$, households invest equally in all factors irrespective of their relative capital gains.

4.4. The intertemporal optimization decision of households

Each region $r$ is assumed to have $h_r$ households that differ in income and wealth. This permits modeling of various income tax structures, as well as examination of the impact of tax policies on different income groups. Households optimize their intertemporal consumption and savings decisions according to a nested Cobb-Douglas/CES utility function, which is widely used in AGE models, subject to an intertemporal budget constraint.

Households have a planning horizon of $\tau$ periods, where $\tau$ is a parameter of the model. At the first level of optimization, households decide how much of their assets and income they want to consume during the next $\tau$ periods, and how much they plan to save in the last period. This part of the utility function is assumed to be Cobb-Douglas, so that

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24 It would be possible to divide households not according to income and wealth but according to age. While a division by age might describe the behavioral differences among consumer groups better than a division by income, the data necessary for the implementation of such a model of the U.S. economy are not available.

25 To simplify notation, the region- and household-specific subscripts $r$ and $h_r$ is ignored in this subsection.
The assumption that the amount of the household’s savings in the final period is an argument of the household’s utility function is the so-called ‘closing condition,’ and ensures that households do not consume all of their wealth in the last period. Although we acknowledge that this assumption is somewhat unsatisfying, it is made in every dynamic AGE model that has a finite planning horizon. We are not aware of alternatives that are computationally feasible for a large number of prices in each period, and at the same time preserve the internal coherence of the model.

\[
U = C^\alpha S_\tau^{1-\alpha}
\]  \tag{9}

where \( C \) is total lifetime consumption, and \( S_\tau \) is saving in the last period. The household’s intertemporal budget constraint is given by

\[
P_c C + \frac{P_{S_\tau} S_\tau}{D_\tau} = P_{T_1} T_1 + P_{S_1} S_1 + P_{M_1} M_1 + P_{L_1} L_1 + TR_1
\]

\[
+ \frac{P_{L_2} L_2 + TR_2}{D_2} + \frac{P_{L_3} L_3 + TR_3}{D_3} + \ldots + \frac{P_{L_\tau} L_\tau + TR_\tau}{D_\tau},
\]  \tag{10}

where \( D_\tau, t = 1..\tau \), is the discount factor from period \( t \) to period 1, \( T_1 \) is the amount of land, \( S_1 \) is the number of structures, and \( M_1 \) is the number of machines that the household owns at the beginning of period 1, \( P_{T_1} \), \( P_{S_1} \), and \( P_{M_1} \) are the period 1 selling prices of land, structures, and machines, respectively, \( P_{L_1} \) is the price of labor (the wage) in period \( t \), \( L_1 \) is the household’s endowment of labor in period \( t \), and \( TR_1 \) is the sum of transfer payments from state and local governments that the household receives in period \( t \). \( P_c \) is a price index of total lifetime consumption.

After having decided how much to save in period \( \tau \), and how much to consume within the next \( \tau \) periods, the household allocates these funds to single period consumption according to

\[
C = (a_1 C_1^\beta + a_2 C_2^\beta + \ldots + a_\tau C_\tau^\beta)^{\frac{1}{\beta}}
\]  \tag{11}

where \( C_t \) is consumption in period \( t \), and \( a_t \) determines the relative consumption share in each period, with \( \sum a_t = 1 \). The elasticity of substitution between consumption in any two periods is given by \( 1/(1-\beta) \). Within each period, the household makes a decision of how much of its funds to spend on consumption, \( C_{tx} \), and how much to spend on leisure, \( \Lambda_t \), using

\[
C_t = (b C_{tx}^\gamma + (1-b) \Lambda_t^\gamma)^{\frac{1}{\gamma}}.
\]  \tag{12}
Finally, the household decides how to allocate consumption expenditure in period $t$ to the individual consumption goods, land, and structures; utility from the consumption of $G$ consumption goods, land, and structures is determined as

$$C_{t} = \prod_{g=1}^{G+2} x_{tg}^{\gamma_{g}}$$

where $x_{tg}$ is consumption of good $g$, and $\gamma_{g}$ is the consumption share of good $g$, with $\sum\gamma_{g} = 1$.\(^{27}\)

If households are able to dispose of their initial assets, it is possible that total consumption of all households will exceed their total income in a given period. Since physical assets are not suitable for consumption, an economy can consume more than it produces only if it is able to import additional consumption goods from producers outside the economy. To permit the economy to dissave beyond the rate of depreciation, the model assumes that foreign producers are always willing to supply consumption goods at domestic net prices in exchange for domestic assets.\(^{28}\)

4.5. The migration decision of households

In addition to solving their intertemporal optimization problems, households must also decide whether to move between periods. The model assumes that households are interregionally and intersectorally immobile within each period, but that they can choose to live in a new region and to work in a different sector after each period. Because the prices that households use in their intertemporal optimization problem are region-specific, both decisions must be made simultaneously.

The optimal solution to this problem is to have each household calculate its utility for movements into all regions and all sectors, and to migrate into the region/sector combination that yields the highest utility. Assuming that the model consists of $r$ regions, $s$ sectors per region, $h$ different income groups per region, members from each of which work in all $s$ sectors, and $\tau$ time periods, this will amount to $h (r s)^{\tau}$ optimizations, which is feasible only for small models, or once computing power has become sufficiently cheap. For the current model we therefore make two simplifying assumptions.

First, we assume that households are able to move only once within the following $\tau$ periods, which reduces the number of optimizations to $h (r s)^{2}$. Second, we assume that households from all regions that are in the same income group will find the same regions and sectors equally desirable. This permits restricting the number of optimizations over all regions and sectors to the number of different income groups in our model,

\(^{27}\) A detailed description of how to solve such a nested utility structure for a single period is provided in Ballard, Fullerton, Shoven and Whalley (1985), Ch. 3.

\(^{28}\) This assumption can only be avoided if the model is extended to explicitly incorporate international trade. Since our current focus is on regional policies, we leave this extension for future development.
which means that only $h \times r \times s$ optimizations per iteration are necessary. This simplification presumably implies that some households do not maximize their intertemporal utilities according to the specified nested utility function. However, the specified utility structure is a simplification of an actual utility structure, which is likely to depend on unobservable factors such as household specific preferences for different regions and sectors. The existence of such unobservable factors seems adequate to explain the fact that some households do not make the migration decision that would optimize their utility as specified within the model. If the necessary computational power becomes available, this assumption can be avoided.

Within each income group the migration decision is determined as follows: all region/sector combinations that provide a utility higher than the mean combination will experience net immigration in the following period, and all region/sector combinations that provide a utility lower than the mean combination will experience net emigration in the following period. To determine which percentage will emigrate from each region/sector combination that provides lower than mean utility, the emigration share $e_j$ for each combination $j < \text{mean}$ is determined according to

$$e_j = \frac{(U_{\text{mean}} - U_j)POP_j}{\sum_{k=1}^{29} (U_{\text{mean}} - U_k)POP_k},$$

(14)

where $U_{\text{mean}}$ is the utility provided by the mean region/sector combination, $POP_j$ is the size of the labor force and $U_j$ is the utility of the region/sector combination for which the emigration share is being determined. The immigration share $i_j$ for each combination $j > \text{mean}$, according to

$$i_j = \frac{(U_j - U_{\text{mean}})POP_j}{\sum_{k=\text{mean}}^{29} (U_k - U_{\text{mean}})POP_k}.$$

(15)

This migration model makes the migration decision utility dependent and at the same time avoids having all households seek to move into a single region and sector. Very few regional AGE models assume that labor is imperfectly mobile. The static model by Morgan, Mutti, and Partridge (1989) and the dynamic model by Feltenstein (1997) contain migration decisions that are functions of the relative wages of regions.29 These solutions permit relative wages to differ among regions while avoiding immediate migration of all persons into one region, but they do not capture the fact that relative wages are only one of the many factors that determine migration. The static models by John Whalley (Whalley and Trela, 1986; Jones and Whalley, 1988,

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29 Morgan, Mutti and Partridge (1989) assume that the migration elasticity of the real wage gap is 0.1, and Feltenstein (1997) uses a logarithmic function of relative wages.
We assume that households and firms consider federally and regionally provided infrastructure to be identical. If sufficient data are available, the model can be extended to make the two forms of infrastructure different from each other. This is the method used by Ballard, Fullerton, Shoven, and Whalley (1985).

4.6. Governments

The government structure consists of one regional government per region, and one federal government. One industry sector in each region produces private goods that are provided by a government agency, for example postal services, and this sector is assumed to charge consumers cost-minimizing prices for its output. Each government also provides a public good (infrastructure) that is produced with all five factors; labor supply for both infrastructure and government-provided private goods is assumed to come from a single pool of workers. Regional governments provide infrastructure only within their own regions, while the federal government provides infrastructure in all regions. All governments are able make monetary transfers to households, and the federal government can also make monetary transfers to any of the regional governments.

To pay for the production of public goods, all governments can levy taxes on the use of any of the four privately owned factors, as well as sales taxes and taxes on the property of households. If a government’s expenditure exceeds its revenue, it can finance the deficit by borrowing at the rate of return to assets. All government debt is repaid in full at the beginning of every period; the repayment of government debt is counted as government expenditure.

Each government is assumed to decide how much of the public good to provide according to a government utility function. However, a government does not maximize government utility subject to a given amount of government expenditure; instead, all governments minimize expenditure, given an exogenously determined level of utility. This has the advantage of permitting a better evaluation of the requirements of counterfactual economic policies. Often the effects of a reduction in the tax on one factor are evaluated under the requirement that the government imposes a different tax on another factor so that government revenue remains unchanged. Despite their intuitive appeal, requirements of this sort do not model a government’s need for funds accurately. If relative prices will change as a result of the reduction in the tax on one factor, the amount that the government collected in taxes before the tax change is irrelevant; what is relevant is the amount that the government will need if it funds the same projects as before, and this amount may be either higher or lower.

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30 We assume that households and firms consider federally and regionally provided infrastructure to be identical. If sufficient data are available, the model can be extended to make the two forms of infrastructure different from each other.

31 This is the method used by Ballard, Fullerton, Shoven, and Whalley (1985).
than the previously collected government revenue. The assumption that a government must retain the same level of ‘utility’ under all alternative tax programs captures the effect of changes in relative prices on the government’s need for funds, and yields a result that is more relevant for the comparison of alternative policies.

4.7. The number of time periods

It is desirable to have a model that solves simultaneously for an economy with many time periods, because the impact of the closing condition on earlier periods decreases as the time horizon increases. The model by Auerbach and Kotlikoff (1987), for example, assumes a time horizon of 150 periods. However, their model is a national model with a single region, a single industry, and 55 (overlapping) household groups, and the calculations during each single iteration can be done extremely quickly; in addition, the search algorithm only must solve for seven variables per period. A model that incorporates $r$ states, each with its own taxes, machines that are perfectly mobile across states, state specific structures and land, and $s$ sectors with industry specific labor needs to determine $r+r+s+2$ unknowns per period. Solution time increases exponentially with the number of prices, and convergence can become extremely slow in even a three period model. We therefore decided to initially restrict the number of periods to two, although the algorithm is able to incorporate any number of periods for which agents have perfect foresight. Once available computing power has increased sufficiently, the model can be extended to incorporate more than two periods.

Two-period models are unable to capture the long-run effects of many policies. The span of time of the model can be extended to additional periods with the following mechanism: all agents are assumed to make their intertemporal decisions with a two-period horizon. At any point in time, agents have perfect foresight over the next two periods and form guesses about the development of prices afterwards. These guesses affect their decisions about how much to save at the end of the second period. At the beginning of period one, all agents determine their actual actions in period one, and their planned actions for period two. At the beginning of period two, all agents have perfect foresight over periods two and three, which permits them to determine their actual actions in period two, and their planned action for period three. Since they now have better information about prices in period three, they change their actual actions for period two compared to what they had planned at the beginning of period one, when only information about periods one and two was available. At the beginning of period three, this process is repeated for periods three and four, and so on.

This method has the advantage of approximating the decision-making process under uncertainty, while at the same time permitting true intertemporal optimization within a relatively simple framework of perfect foresight for a limited span of time into the future. If sufficient computing power is available, it is straightforward to extend the method from two periods of perfect foresight to any number of periods of perfect foresight.
4.8. General Equilibrium for the dynamic two-period model

For each iteration, the algorithm determines the differences between quantity demanded and supplied ('excess demand') for all factors. This proceeds as follows:

1. Choose a new set of prices for all factors for both periods.\(^{32}\)
2. Given factor prices and production functions, determine output prices for all sectors by minimizing production cost.
3. Calculate the sale value of all assets and determine the rate of return to investment.
4. Determine government expenditures on public goods and government transfers to households.
5. Calculate households’ assets and incomes, and determine migration, demand for consumption goods, and savings in each period.
6. Given household savings, determine the allocation of savings to investment in assets and to payment for government debt.

   The algorithm has a simultaneity problem at this point. While all government expenditures are known at this point, government revenue (and therefore government deficit) will be known only after factor demand by all sectors in all regions is known, because factor tax rates may differ across sectors and regions. Factor demand can be determined only after it is known how many more buildings and machines need to be constructed, which in turn can be decided only after the size of the government deficit, which competes for the same funds as asset investment, is known.

   The simultaneity problem is solved by introducing an additional ‘price’ into the price vector, which represents a guess of the size of government revenue during each period, and is used to guess the size of the deficit before the actual government revenue can be calculated. Differences between the guessed revenue and the actual revenue are treated the same way as differences between quantities demanded and supplied for all factors, and the convergence algorithm adjusts this extra price so that, in equilibrium, the guessed revenue and the actual revenue are identical.\(^{33}\)

7. Determine total demand for consumption goods and government revenue.
8. Determine excess demand for all factors in all periods.
9. If the sizes of all excess demands are below the required threshold, an equilibrium solution has been found. Otherwise, proceed with 1.

---

\(^{32}\) To find the equilibrium we use a combination of Merrill’s algorithm and Broyden’s method (Broyden, 1965). Merrill’s algorithm, which is based on Scarf’s fixed-point method, has the advantage that it guarantees convergence to the equilibrium from any combination of starting values, but unfortunately it is rather slow. Broyden’s algorithm is a quasi-Newton method that uses numerical approximations of the Jacobian matrix; from a good starting vector, its convergence to the equilibrium is very fast, but from a bad starting vector, the algorithm might not converge at all (the often-used Gauss-Seidel method has similar difficulties). We obtained the overall fastest reliable convergence rate by using Merrill’s algorithm to find a suitable starting vector, and Broyden’s method to solve the model up to the desired precision.

\(^{33}\) See Shoven and Whalley (1973).
5. Simulation of a policy change in a two-period, three-region, four-industry model

To indicate that the model described in the previous section generates results that go beyond those that are possible with existing regional models, we used our model to simulate the result of a policy change in a hypothetical economy with three regions, and four industries and two consumer groups per region. In every region, each of the two consumer groups has a total labor endowment of 8,000 efficiency units, which are divided equally among the four industries, and in each region Group 1 owns 13.33 percent and Group 2 owns 20 percent of the assets of all regions. Each region has a total supply of 3,000 units of land, 3,000 structures, and 1000 units of infrastructure, and the total supply of machines for all three regions is 6,000 units. Industries 1 and 2 produce two consumption goods, Industry 3 is the construction industry that produces new structures, and Industry 4 produces private goods that are supplied by the regional governments (e.g. postal service); labor with Industry 4 skills is also used by the regional governments to produce infrastructure.

Columns 1 and 2 in Table 2 show the ‘benchmark case’ in which the governments of all regions have adopted identical tax and expenditure policies. Each regional government imposes corporate income taxes of 20 percent on the return to the use of land, structures, and machines, a 20 percent tax on the sales of consumption goods, a 20 percent tax on personal labor and asset income, and each regional government spends its revenue on 358.7 units of infrastructure (the number was chosen to keep the first period regional budgets roughly balanced). For simplicity we assume that no government makes transfer payments to any consumer group, and the federal government neither imposes taxes nor provides (global) infrastructure. Machines are assumed to depreciate at a rate of 7 percent per period, structures at a rate of 5 percent, and infrastructure at a rate of 35.87 percent (i.e. the total supply of infrastructure remains constant in both periods).

The model parameters are chosen so that the benchmark economy is close to a steady state—investment in machines and structures is just high enough to replace depreciated capital, and net investment is very low; all jobs pay a very similar wage so that labor migration between industries and regions is minimal. Table 2 shows the real interest rate, the supply of machines for the economy, the supply of land and structures, the labor supply to and the output of each of the four industries in each region, and the budget deficit of each regional government.

As an alternative to the ‘benchmark case’ we assume that Region 1 wants to provide an incentive for new construction, and lowers the tax rate on income from structures from 20 percent to 10 percent. This is similar to policies enacted by many communities in the US that seek to increase construction within their borders by providing tax relief for new structures. To keep the budget deficit unchanged compared to the ‘benchmark case,’ the tax rate on labor is increased from 20 percent to 21.875 percent.

The effects of this policy change are shown in Columns 3 and 4. As a result of the reduction in the tax on the services of structures, net investment in structures in Region 1 increases from 16.3 to 76.7 units. Since the real interest rate increases only slightly and total savings remain almost unchanged, this increase in
construction comes at the expense of investment in structures in the two other regions, where net investment falls from 16.3 to 0.3 units, and at the expense of investment in machines, which falls from 98.1 to 37.9 units. Although construction has increased in Region 1, the net effect of the policy change on production in Region 1 is negative, because of the corresponding increase in the personal income tax on labor income, which makes work in Region 1 less attractive; all four industries in Region 1 experience outflows of workers.

The other two regions experience reductions in the net increase of the number of structures, which reduces the productivity of the other factors of production in these regions. This leads to a reduction of labor supply in these regions as well as to a reduction of output in all industries except the construction industries (which produce structures that will be used in Region 1). Since total industry output in Industries 1, 2 and 4 decreases by more than the increase in output of the construction industry, and because output prices of all industries in any given period (not shown in the table) are almost identical, the net effect on the economy’s GDP is negative.

Besides the results that are shown in Table 2, the simulation provided information about which consumer group in which region benefitted/lost most from the policy change in terms of changes in wealth, income and utility, about changes in each consumer group’s consumption, labor supply, and saving behavior, about changes in relative wages across industries and regions, about changes in relative sale values of machines, structures, and land, and about changes in the composition of government demand that are due to changes in relative prices of factors and goods that the government purchases to produce infrastructure.

6. Summary and Conclusion

This paper has motivated the use of AGE models that are (1) dynamic, (2) incorporate the sale value of assets to describe the welfare effect in the household’s utility-maximization problem and to derive the return to investment endogenously, (3) model land, labor, structures, machines and infrastructure as five different factors of production, and (4) treat identical goods that are produced in different regions as perfect substitutes. Existing regional AGE models generally do not possess these properties and no previous model possesses all of them. Although these properties cause the model and the algorithm to be more complicated, we consider them essential for a general equilibrium analysis of the effects of regional economic policies.

We describe a model that has the four properties, and report the results of a simulation of a three-region economy as evidence that our model generates interesting results.
Table 1. Properties of Existing Regional Applied General Equilibrium Models

<table>
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<tr>
<th>Author(s)</th>
<th>Date of Publication</th>
<th># of Periods</th>
<th># of Regions</th>
<th># of Sectors</th>
<th>Labor mobility</th>
<th>Capital mobility</th>
<th>Land mobility</th>
<th>Public goods</th>
<th>Input Output matrix</th>
<th>Saving</th>
<th>Balanced Budget</th>
<th>Non-traded goods</th>
<th>Transportation Cost</th>
<th>Money</th>
<th>Model Application</th>
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<td>Kimbell and Harrison</td>
<td>1984</td>
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<td>2</td>
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<td>IS MR</td>
<td>MS, MR and MS, IR</td>
<td>IS IR</td>
<td>-</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Four-factor framework with a two-factor application to tax policy in California</td>
</tr>
<tr>
<td>Depotakis and Fisher</td>
<td>1988</td>
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Notation:  
1. MS = perfectly mobile across sectors; PS = partially mobile across sectors; IS = immobile across sectors; MR = perfectly mobile across regions; PR = partially mobile across regions; IR = immobile across regions; PI = supplied perfectly inelastically; E = supplied elastically; ‘-‘ = not included.  
2. An ‘N’ in the column ‘Balanced Budget’ indicates that the size of the budget deficit is exogenously determined.  
3. An ‘N’ in the column ‘Saving’ indicates that households spend their whole income on consumption goods.  
4. A model is denoted as incorporating ‘Public Goods’ only if the availability of public goods affects either a sector’s ability to produce, or a households’s consumption or location decision, and a model is denoted as not incorporating ‘Public Goods’ if public goods are either not modeled or introduced with the only purpose of disposing of tax revenue. ‘Public Goods’ are denoted as ‘supplied inelastically’ if it is assumed in the paper that the amount of public good that is supplied does not change in the counterfactual analysis.

Note:  
1. The table shows only models with an explicit multi-regional structure; models that draw regional inferences from one-sector national models (‘top-down models’) are not included.  
2. All models in this table (with the exception of the framework by Kimbell and Harrison) use the Armington assumption that goods that are produced in different regions but are otherwise identical are imperfect substitutes for each other. The model by Kimbell and Harrison assumes that consumers and local governments consume only goods that were produced within their region of residence.  
3. The framework by Kimbell and Harrison is the only one that includes both mobile capital and immobile capital (although their application to California uses only mobile capital).  
4. None of the models differentiates between the sale price and the rental price of a factor of production.
Table 2. Simulation Results of a Policy Change in a Hypothetical 3-Region Economy

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<td>0.07%</td>
<td>3,329.3</td>
<td>3,321.3</td>
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<td>1,052.9</td>
<td>1045.5</td>
<td>-</td>
<td>1,048.3</td>
<td>1,041.3</td>
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<td>3,329.3</td>
<td>3,321.3</td>
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<tr>
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<td>1045.5</td>
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<td>1,048.3</td>
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<td>-</td>
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* Labor is measured as the product of the number of workers and hours supplied per worker.
References


