Math 227 Sample Final Examination 1

Name (print)_____

Name (sign)_____

Bing ID number _____

(Your instructor may check your ID during or after the test)

No books, notes, or electronic devices (calculators, cell phones, smart watches, etc.) are allowed. Write all your work on the test – nothing else will be graded. **You must show all your work to receive credit.** Your work must be legible, and the final answers must be reasonably simplified.

On some problems you may be asked to use a specific method to solve the problem (for instance, "Use the power series method to find..."). On all other problems, you may use any method we have covered. **You may not use methods that we have not covered.**

Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

Problem 1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n}{n}(x-2)^n$$

Problem 2. (10 points) Use the power series method to find the limit .

$$\lim_{x\to 0} \frac{xe^{x^2} - \sin x}{\arctan(2x^3)}$$

Problem 3. (5 points) Suppose $\sum_{n=0}^{\infty} a_n(x-4)^n$ converges conditionally at $x = 15$. What is the radius of convergence of the series $\sum_{n=0}^{\infty} 2^n a_n(x-3)^n$?

Problem 4. (20 points) The Binomial Series formula can be used to show that

$$(1+x)^{-1/2} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{4^n (n!)^2} x^n$$

a) Find the radius of convergence of this series.

b) Use this formula to obtain the Maclaurin series for $\frac{1}{\sqrt{1-x^2}}$

c) Use the result of part b) to obtain the Maclaurin series for $\arcsin x$.

d) Use the result of part c) to find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 \cdot 16^n \cdot (2n+1)}$$

Problem 5. (10 points)
a) Find the Maclaurin series for $f(x) = \cos(2x^3)$

b) Find $f^{(12)}(0)$

Problem 6. (10 points) Find the sum of the series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^n}{4^n \cdot n!}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \cdot 3^n}{4^n \cdot (2n)!}$

Problem 7. (20 points)
a) Find the Maclaurin series for e^{-x^2}

b) Find the Maclaurin series for $\int e^{-x^2}\ dx$

c) Find $\int_0^{1/2} e^{-x^2}\ dx$ as an infinite series.

d) How many terms of the infinite series from part c) need to be added to approximate the integral accurately to within 10^{-6}? Use the Alternating Series Error Estimation theorem to justify. (You do not need to calculate the approximation. You may use $2^{10} = 1024$.)

Problem 8. (10 points) Find the Taylor polynomial $T_3(x)$ centered at $a = 4$ for the function

$$f(x) = \sqrt{2x + 1}$$

Problem 9. (10 points)

a) Write down the linear approximation to the function $f(x) = \sqrt{x}$ at $a = 9$.

b) Approximate $\sqrt{11}$ using this approximation.

c) Use the Taylor's Inequality with the best possible constant M to estimate the error of this approximation.

Math 227 Sample Final Examination 2

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Problem 1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{3^n}{(n^2-n)}(2x-1)^n$$

Problem 2. (10 points) Use the power series method to find the limit .

$$\lim_{x\to 0} \frac{x-\sin x}{1+\ln(1+2x)-\cos(2x)-2x}$$

Problem 3. (5 points) Suppose $\lim_{n\to\infty} a_n = 20$. Find the center and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{a_n}{5^n}(x-16)^n$$

Problem 4. (15 points)
a) Find the Maclaurin series for the function

$$f(x) = \frac{1}{x-1}$$

b) Find the Maclaurin series for the function

$$g(x) = \frac{1}{x-2}$$

c) Find the Maclaurin series for the function

$$h(x) = \frac{1}{(x-1)(x-2)}$$

(Hint: use partial fractions to relate $h(x)$ to $f(x)$ and $g(x)$.)

Problem 5. (10 points)
Find $T_4(x)$ centered at $a = 0$ for the function

$$f(x) = \sin(2x) \cdot \cos(3x)$$

Problem 6. (10 points)
a) Write down the quadratic approximation to the function $f(x) = x^{3/2}$ at $a = 4$.

b) Approximate $\sqrt{4.4}$ using this approximation.

c) Use Taylor's Inequality with the best possible constant M to estimate the error of this approximation.

Problem 7. (20 points)
a) Find the Maclaurin series for $x^4 \cos 3x^2$

b) Find the Maclaurin series for $\int x^4 \cos 3x^2 \, dx$

c) Find $\int_0^2 x^4 \cos 3x^2 \, dx$ as an infinite series.

Problem 8. (10 points) Find the Taylor polynomial $T_3(x)$ centered at $a = 1$ for the function

$$f(x) = \arctan x$$

Problem 9. (10 points) Find the sum of the series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{n!}$

Math 227 Sample Final Examination 3

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Problem 1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(n^2-n)}{3^n}(2x+1)^n$$

Problem 2. (15 points) Use the power series method to find the limit .

$$\lim_{x\to 0} \frac{x \arctan x - \sin(x^2)}{\cos x \cdot \sqrt{1+x^2} - 1}$$

Problem 3. (5 points) Suppose $\lim_{n\to\infty} 2^n \cdot a_n = 3$. Find the center and the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{a_n}{5^n}(2x+7)^n$.

Problem 4. (20 points)

a) Find the Maclaurin series for the function

$$f(x) = \frac{1}{1+x}$$

b) Use differentiation of power series and the result of part a) to find the Maclaurin series for the function

$$g(x) = \frac{1}{(x+1)^2}$$

c) Use differentiation of power series and the result of part b) to find the Maclaurin series for the function

$$h(x) = \frac{1}{(x+1)^3}$$

d) Find the sum of the series

$$\sum_{n=3}^{\infty} \frac{n(n-1)}{2^n}$$

Problem 5. (10 points)
Find $T_3(x)$ centered at $a = 1$ for the function

$$f(x) = e^{x^2}$$

Problem 6. (15 points)
a) Use the Maclauren series for e^x to represent $e^{0.1}$ as an infinite series.

b) Find $T_2(0.1)$, where $T_2(x)$ is the degree 2 Taylor polynomial for the function e^x at $a = 0$. What partial sum of the series from part a) does it correspond to?

c) Use Taylor's Inequality to find the smallest n such that the Taylor polynomial $T_n(x)$ approximates e^x at $x = 0.1$ with the error less than 10^{-6}. Hint: You may use the fact that $e^{0.1} < 2$.

Problem 7. (15 points)
a) Find the Maclaurin series for

$$x \arctan(x^3)$$

b) Find the Maclaurin series for

$$\int x \arctan(x^3)\ dx$$

c) Express as an infinite series

$$\int_0^{1/2} x \arctan(x^3)\ dx$$

Problem 8. (10 points) Find the sum of the series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-\pi^2)^n}{(2n+1)!}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$