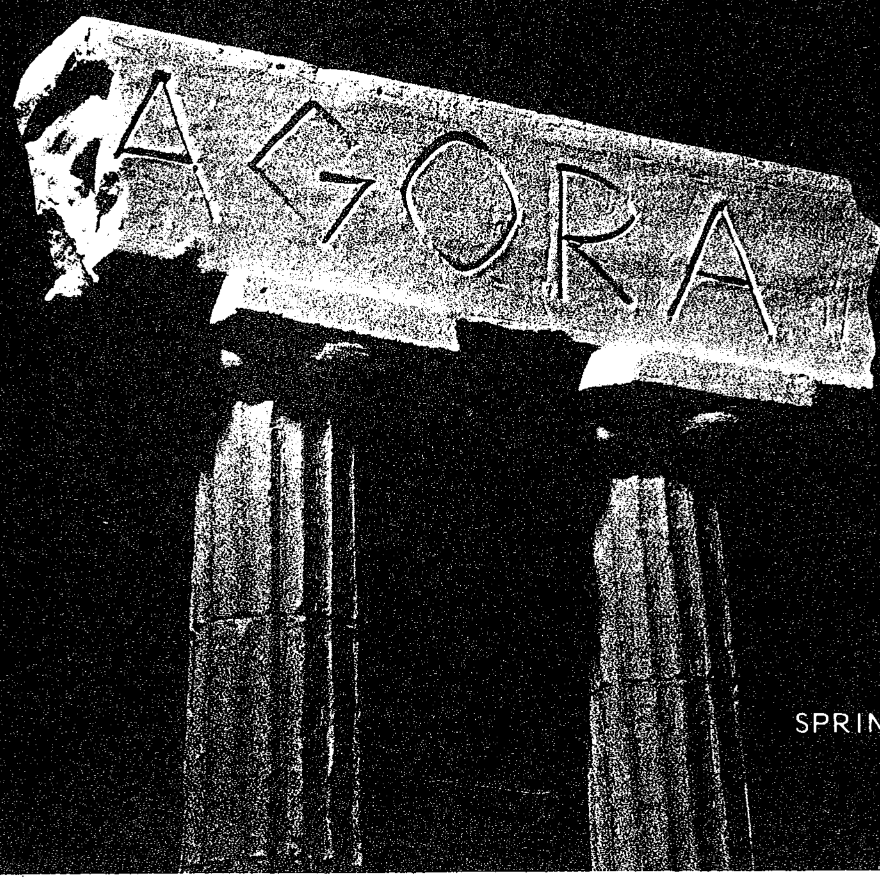
AGORA A JOURNAL IN THE HUMANITIES AND SOCIAL SCIENCES



SPRING 1970

AGORA is published at The State University of New York College at Potsdam in the Fall and Spring. Subscription rate: $2.50 for two issues; $1.50 for single issues; $4.00 for annual institutional rate. Correspondence should be addressed to: Martin A. Bertman, Philosophy Department, or Onnik K. Keshishian, History and Political Science Department, AGORA, SUNY, Potsdam, N. Y. 13676.

Spring 1970

VOLUME 1
NUMBER 2

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BOOK REVIEWS

The Continuous Analogy: The Uses of Continuous Proportions in Plato and Aristotle

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Introductory: the historical and philosophical questions.

Analogical reasoning in philosophical arguments has received some attention in recent years; Thomistic philosophers are concerned with such arguments in theological contexts,[1] but philosophers of science too have come to find many important examples of analogical arguments in their areas of investigation.[2] Historians of philosophy are discovering that one can find relationships between various philosophers by an investigation of the kinds of analogies which they use.[3] It may be that all these investigations will lead to an advance in philosophical knowledge, for we learn which sorts of arguments turn out well and which badly.

Analogical arguments are either fundamentally mathematical in nature, or find their most precise examples in mathematical proportions. Both Plato and Aristotle explicitly consider mathematics as the basis of the validity of analogical arguments; to some extent, we can discover in their respective uses of analogical arguments their understanding of the relationships between mathematics and the world. Plato's use of continuous proportions and analogies in general and in principle are philosophically correct, and certain of Aristotle's uses of continuous analogies are in principle wrong, and philosophically pernicious.

I. The meaning of 'analogy' in Plato and Aristotle.

Recent investigations of analogical arguments have usually concentrated on just one of the forms of analogical argument, the discrete analogy.[4] Some writers on Plato have, however, recognized the importance of the continuous analogy, at least as it appears in **Timaeus** 32 b.[5]

A. The forms of analogy, as they appear in Plato and Aristotle.

Plato does not explicitly name the forms of analogy which he uses, but Aristotle does name them in **Nicomachean Ethics** V. 3-4, 1131a31-1132b20. Aristotle here distinguishes 'geometrical' and 'arithmetical' analogies, and finds two forms of 'geometrical' analogy, the discrete and the continuous.[6] The names 'geometrical' and 'arithmetical' immediately call attention to the mathematical source of the term **analogia**; Plato usually and Aristotle sometimes use the word in the mathematical sense of 'proportion', although perhaps not precisely with the same connotations it might have for a modern mathematician. The 'arithmetical' analogy or proportion looks least like a proportion to us: its form, as described in **NE** V.4, is a-b c-d, for example, 6-3 4-1. Aristotle uses this form of analogy in his discussion of rectificatory justice, and rarely elsewhere. Plato uses the arithmetic mean, which is derived from an arithmetic proportion, at **Timaeus** 36a; in the same dialogue he also uses an harmonic mean, which implies an harmonic proportion.[8]

We can derive from Aristotle's description in **NE** V.3 the two forms of geometrical analogy thus: the discrete analogy is a:b::c:d; the continuous analogy is a:b::b:c. 1:2::3:6 would be an example of a discrete analogy or proportion; 1:2::2:4 would be an example of the continuous analogy or proportion. Aristotle suggests in **NE** V.3 that the continuous analogy is reducible to the discrete analogy, in that the discrete analogy has four terms explicitly, and the continuous has four terms implicitly, since the middle term is one term used as two. However, there are important differences between the discrete and continuous forms of analogy in actual use. Plato, at **Timaeus** 32, seems to suppose that the continuous analogy is fundamental.

> Of all bonds the best is that which makes itself and the terms it connects a unity in the fullest sense; and it is of the nature of a continued geometrical proportion to effect this most perfectly. For whenever, of three numbers, the middle one between any two that are either solids or squares is such that, as the first is to it, so is it to the last, and conversely as the last is to the middle, so is the middle to the first, then since the middle becomes first and last, and again the last and first become middle, in that way all will necessarily come to play the same part towards one another, and by so doing they will all make a unity.[9]

To be sure, philosophical writers do tend to use discrete analogies somewhat more frequently, at any rate with the name 'analogy' in mind. Some examples of discrete analogies from ancient writers will help to make the contrast clearer: "Hair and leaves and thick feathers of birds are the same thing in origin, and reptiles' scales too, on strong limbs;"[10] "As healthy and diseased conditions are to the body, so are acting justly and acting unjustly to the soul;"[11] "Just as the saw exists for sawing... so the body exists for the soul."[12] These analogies, and others like them, are justly famous as philosophical arguments.

I. The types of continuous analogy.

Plato and Aristotle use at least two forms of the continuous analogy; one has the name **taxis**[13] applied to it—we might understand this as a 'closed set'; the other I shall call the 'open-ended' form. In the **taxis**, all the members are clearly stated or easily available, and are all open to investigation; there is a clear beginning point and a clear end point. In the open-ended form, however, the continuous analogy is used to generate possible members, some of which may not be available to investigation and either the beginning point or the end point, or both, are theoretical in character; the basis of the claim that they exist depends largely upon the force of the analogy itself.

I.B.1. The **taxis.**

a) Aristotle's bees.

An example will, I hope, clarify the character of the type of continuous analogy called the **taxis**. In stating his conclusions concerning the generation of bees,[14] Aristotle says that the leader (the queen) generates its own kind and also the 'bees' (workers), and that the 'bees' do not generate their own kind but do generate drones, and that the drones do not generate. In the form of an analogy, this may be stated: leader (queen) : bee (worker) :: bee : drone.[15] A mathematical proportion is suggested, not in terms of the size of the kinds of bees, as some reader interpolated (760a13ff), but in terms of the relative power to generate, as Peck notes **ad loc.**[16] Aristotle says that the leaders have twice the generative power of the bees, so the bees should have twice the generative power of the drones. The drones, one may object, generate nothing, implying no generative power; however, nutritive power is a form of generative power, and drones do grow. If the nutritive power of the drones is arbitrarily assigned the value '1', then the bees may be said to have twice that, or '2'; the leaders could be said to have '4'.

In this example, we can see that a **taxis** exhaustively enumerates the possible members of the analogy within the analogy as stated, and that there is no temptation to hypothesize further members, such as something having twice the generative power of the leaders. The **taxis** is an essentially closed but ordered set.

b) Aristotle's Cosmos

The primitive examples, or paradigm cases, of series of moving causes may be stated as continuous analogies in the **taxis** form. For example, 'the carpenter moves the tools, and the tools move the wood.'[17] In this case the carpenter is the source of movement, and that which is made out of the wood is the end of the movement. In the biological books, as in the ethical books, Aristotle supposes, if I may generalize, that men and animals initiate series of moving causes, and that the products of their actions are the ends of such series. Indeed, in the **Movement of Animals** Aristotle analyzes the physiological process of animal movement in terms of such a **taxis**: if one moves a stick in one's hand, the source of movement is not in the stick, nor in the hand, "for the end of the hand has the same relation to the wrist as the stick has to the hand, and the wrist has this relation to the elbow," and so on (702b2). The end of the argument is that there is some central origin of movement in the body, which Aristotle identified with the heart.

It is even the case that Aristotle attempts to explain the teleological structure of the whole universe as a **taxis**:

> One must consider in what sense the nature of the universe has the good and the best, whether as something separate and independent, or as the **taxis**. Or is it in both ways, as an army? for the good of an army is both the general and in the **taxis**, but more the general; for he does not exist because of the **taxis**, but it exists because of him. All things are ordered together (**syntetaktai**) somehow... everything is ordered together to one end (**pros hen**), as in a household....[18]

In the context, Aristotle does not state very clearly the way in which the members of the **taxis** are ordered together, except to say that they are not all ordered in the same way. One may suppose that he is thinking of something like 'God : heavenly beings : : heavenly beings : man : : man : animals ...'. Two discrete analogues are thrown in for good measure, and each is itself a continuous analogy, so that the whole structure might look something like this:

God : stars, etc. : : stars : men : : men : animals ...
General : officers : : officers : enlisted men : : enlisted men : servants.
Householder : freeman : : freeman : slaves.

The **taxis** or 'order' of the universe is meant to be demonstrated by the continuous analogies which arrange series of 'causes' (in the Aristotelian sense); we will try to show that Aristotle is not successful in showing that the order of the universe is a **taxis**, in that he must use at least two continuous analogies which are essentially open-ended, and that his attempts to close these analogies are failures.

c) Plato **Tim.** 30ff

Plato's uses of continuous analogies in the **taxis** form are usually rather more mathematical in character than Aristotle's. There are to be sure some examples of non-mathematical analogies of this type. One, a commonplace of ancient thought, is this: as cattle are the possession of men, so men are the possession of the gods;[19] another is more peculiar to him: as mirror images are to the objects in the world, so such objects are to the forms.[20] In both cases the 'top' and 'bottom' of the analogy are clearly stated—the gods are owned by no one, and the cattle own nothing; and even if one were to insert 'mathematics' into the metaphysical example, the set would still be closed at top and bottom by the forms and the mirror images.

At **Timaeus** 30ff there is a continuous proportion, which seems to be both a **taxis** and mathematical. We have already noted the general theory of the passage; it is applied first to the four elements: because two means are required for a three dimensional world,

> the god set water and air between fire and earth, and made them, so far as was possible, proportional to one another, so that as fire is to air, so is air to water, and as air to water, so is water to earth, and

> thus he bound together the frame of a world visible and tangible. ...coming into concord by means of proportion, and from these it acquired Amity, so that coming into unity with itself it became indissoluable by any other save him who bound it together. (32, Cornford translation)

Cornford has a most interesting account of the possible meanings of this continuous analogy but it is disappointing at last since he does not decide just how Plato intended the quantitative analysis to apply to these elements. At 53c. Plato returns to the subject of the elements, constructing the first four of the five regular solids out of two sorts of right triangles. It is clear that Plato means to construct a continuous proportion or analogy like this: Fire : air :: air : water :: water : earth. One series which has this characteristic is 8 : 12 : 18 : 27 (each number is 3-2 its predecessor). Later, Plato generates the two cubes 8 and 27 with two continuous analogies: 1 : 2 :: 2 : 4 :: 4 : 8; 1 : 3 :: 3 :x 9 :: 9 : 27. One is the point, 2 and 3 are linear numbers, 4 and 9 are plane numbers (they are squares), and 8 and 27 are solid (or cube) numbers. So it seems that the numbers 12 and 18 are found as the two mean proportionals between 8 and 27, at least this would be an hypothesis which is supported by evidence from the dialogue. The proportion is a **taxis** too, for the extreme terms are found first as the first two perfect cubes and then the middle terms are found as mean proportionals, and it turns out (magically enough) that they are whole numbers. Within the dialogue this means that the generation of the soul of the world precedes as a necessary condition for the generation of the body of the world. For further mathematical analysis of the soul of the world, one might look at the commentary on the **Timaeus** written by Proclus. But not even Proclus manages to consolidate the series 8 : 12 : 18 : 27 with the first four regular solids.

It might, incidentally, be of interest to note that in the **Phaedo**, at 111, there is a continuous analogy, earth : water :: water : air :: air : **aither** (rather than fire, as in the **Timaeus**), and that in the **Epinomis** there is a **five** element theory, adding **aither** after fire, as Aristotle tends to do.

d) Plato, **Republic** VI, 509-511: The Divided Line.

Plato's divided line can be interpreted mathematically in a manner related to the interpretation of the proportion of the elements in the **Timaeus**. First, one must look at a very special example of the continuous analogy, the one which generates the Golden Number: a : b :: b : a b. This proportion uses, you will note, only **two** terms—Aristotle does not seem to have been aware of it, or rather he probably didn't understand it. Plato does seem to have been aware of this proportion, however; it does seem to be implied by the mathematics of **Theaetetus** 147d-148b.[21] It is directly related to (or cause of) the so-called Fibonacci series:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55... (actually, one may start with any two numbers) in which each number, from the third on, is the sum of its two predecessors.

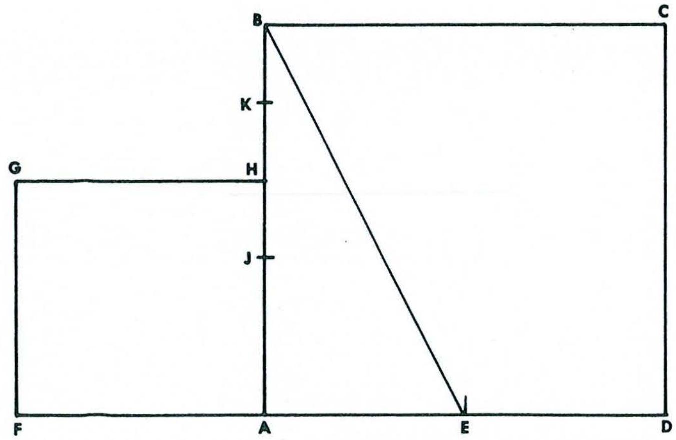
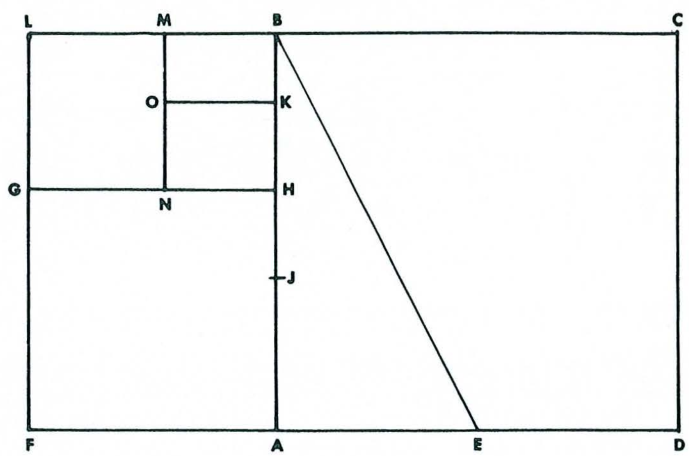


FIGURE 1

AB is the given line. The square ABCD is constructed, and AD is bisected at E by a line drawn from B. AD is extended, and EF is marked off equal to EB. The square AFGH is completed, with H located on AB so that AB : AH : : AH : HB. AJ, equal to HB, is laid off on AB; and HK, equal to JH, on HB. In this way AH and HB are divided in the same ratio as AB. (R. S. Brumbaugh, **op. cit.** app. B, pp. 270-71; this is his figure 108.)

FIGURE 2



If one forms the ratio by putting the successor over its predecessor, e.g. 55-34, one rapidly approximates the Golden Number, or Golden Section $\sqrt{\frac{5+1}{2}} = 1.618\ldots$; This number is known as **Phi** by Ghyka and others. A correlate of this number is formed by putting the predecessor over the successor, e.g. 34-55, rapidly approximating $\sqrt{\frac{5-1}{2}} = .618\ldots$. This number is called **tau** by Thompson, in "Excess and Defect". Both Ghyka and Thompson refer to the use of these numbers, these series, in Euclid II.11, for the construction of regular pentagons, and thence to the construction of the regular dodecahedron. But most importantly for us now is the relationship of this series to the division of a line according to 'mean and extreme ratio'. If the line AC is divided according to mean and extreme ratio, we have three magnitudes, AC, AB, BC, such that $AB^2 = AC \times BC$. Any two Fibonacci numbers approximate the lengths of AB and BC, or to put it another way, if a line ABC is divided according to mean and extreme ratio, then BC-AB = Phi.

Now suppose that one has a rectangle whose sides are equal to AB and BC of line AC divided according to mean and extreme ratio. This rectangle may be known as a Phi rectangle. A Phi rectangle is precisely that figure whose **gnomon** is a square.[22]

Brumbaugh points out, in appendix B of **Plato's Mathematical Imagination**, that the line in **Republic** VI is constructed by means of 'mean and extreme ratio', as in Euclid II.11, His geometrical analysis is given here as 'figure 1'. You will note in his analysis that he has discerned here one continuous proportion, AB : AH :: AH : HB. This continuous proportion may be carried even farther, HB : HK (HJ) :: HK : KB (and AJ HB). Brumbaugh does not make explicit this use of continuous proportion, at least not as explicit as he might have, but he doesn't even suggest the possibility of relating this figure to the Phi rectangle which is so readily constructed (as in figure 2), BHGL. The square GHAF is, of course, its gnomon, generating the further Phi rectangle, BAFL, of which the square BCDA is the gnomon, generating the Phi rectangle CDFL. It would also be the case that MBHN and MBKO form **Phi** rectangles.

I would be foolhardy to suggest that I could demonstrate that this is the one correct interpretation of the line passage, as a geometrical figure. The most that I wish to claim here is that this interpretation is consistent with the text and consonant with Plato's **modus operandi.** It would also make rather more sense than usual of the manner of unification which is possible for the modes of cognition and modes of existence which are organized in terms of segments of the line. But showing this is beyond the scope of this paper, which is (after all) about uses of continuous proportions.

If my interpretation of the geometry of the line is correct, then the line is an example of the use of a continuous proportion or analogy of the **taxis** variety. Although the series of **phi** rectangles and squares is potentially open-ended, the series is closed off by the fact that the states of knowledge (which are discoverable by other means) are all accommodated. If there were some state of cognition higher than **nous**, or lower than **eikasia**, then the process could be carried one more step, but there is no reason to carry it on once all forms of cognition are accounted for. This makes the series a **taxis**. If one likes, one might say that the series is potentially open-ended, but there is no reason to look for further members.

I.B.2. Open-ended continuous analogies.

a) General cases.

In the open-ended continuous analogy, there is no very obvious first member or last member, or perhaps neither is very obvious. 1 : 2 :: 2 : 4 ..., and many similar series, are taken by mathematicians as generative of infinite series; this series is open-ended at one end, but could be made open-ended at both ends by structuring it thus: ... .25 : .5 : 1 :: 1 : 2 ... .

Genealogies, especially those of the biblical sort,[23] are presented in the form of continuous analogies, usually open-ended. A genealogy which ends with a person who died childless is closed at one end. The desire to close a genealogy at the beginning and make it a **taxis** has been a temptation to a variety of "metaphysical" argument. The ancient Greek nobility liked to use some god or other as the starting point for their genealogies; Adam and Eve serve this purpose, in a general way, for several religions. Aristotle, in contrast, seems to suppose that this type of analogy goes back indefinitely, or might go forward indefinitely.[24] If we take him to say that genealogies must go back (or forward) infinitely, this would be an equally 'metaphysical' use of the open-ended continuous analogy. Empirically, we would of course say that the evidence supports the view that possible genealogies go back in time an indefinite, but finite, number of generations, and that the members are not known beyond a much lower number of generations; and that generations forward in time will no doubt be finite in number too.

b) Aristotle's cosmos again.

Aristotle analyzes his 'four causes' in terms of continuous analogies; these analogies are often open-ended, although he would like some of them to be taxeis. We have already noted the teleological arrangement of the world as a continuous analogy, in **Metaphysics** Lamda. The series of materials seems to provide the other end of this, or a related, continuous analogy, and it is actually open-ended:

> It seems that when we call a thing not something else but 'thaten'—e.g., a casket is not 'wood' but 'wooden', and wood is not 'earth' but 'earthen', and again earth will illustrate our point if it is similarly not something else but 'thaten'—that other thing is always potentially (in the full sense of that word) the thing which comes after it in this series. ...And if there is a first thing, which is no longer, in reference to something else, called 'thaten', this is a prime matter... .[25]

The analogy is open-ended because the last actual material named is 'earth', but the possibility is left open that there is some material which is potentially earth, and the possibility is also left open that there are several levels of potential existence, or matters, below the level of earth. To be sure, a last member of the series is named, the famous 'prime matter', but this is a theoretical concept, devised precisely to put a stop to the regress of potentialities.

If one turns the analogy of materials the other way around, it is a continuous analogy of form, rather than of matter; of actuality rather than of potentiality. The series of species and genera which reiterates in larger and larger classes would also be an example of a continuous analogy, possibly open-ended. At any rate, the largest genera as they are named by Aristotle are arranged conceptually, without much regard as to whether there are actual members or not. In **Metaphysics** Epsilon 1, for example, he takes the two concepts 'mobile—immobile' and 'separable—inseparable', and makes a general classification of beings: mobile and inseparable, immobile and inseparable, immobile and separable (mobile and separable beings do not exist); similarly in Lambda, the major division is mobile—immobile, and then the mobile beings are divided into everlasting and perishable. Aristotle does try to put members into each class, but the classification is less than convincing when he tells us that the stars are everlasting, and that there must be at least one immobile and separable being. It is not a classification empirically derived, but rather one which has been theoretically derived to serve the ends of the metaphysical system. The **beginning** of the continuous analogy which 'ends' in these classes is in fact empirically derived: there are good empirical reasons supporting Aristotle's system of classification of animals, for example. But somewhere along the way the method of determining the genera shifts from largely empirical (and somewhat theoretical) to largely theoretical (and very little empirical). As an empirical continuous analogy, the analogy of genera is open-ended.

Although the paradigm cases of series of moving causes are closed analogies or **taxeis**, it is not difficult to make them open-ended. In the case of the carpenter who moves the tools which move the wood, one need only ask, 'what moves the carpenter?' and an indefinite series of questions, each moving back the origin of the series, may follow. Or in the case of the arm — hand — stick example, instead of going back to the 'heart', one might ask about the consequences of the movement of the stick, and this might be pushed forward indefinitely.

A simple example of a continuous analogy of final causes would be: as the organ serves the man, so the man serves the state.[26] This is a **taxis**, as most continuous analogies of final causes would be. The most general form of the continuous analogy of final causes is stated in **Met.** Lambda 10, already noted; God or 'the general' closes off the series. But this analogy of final causes is also co-ordinated with a continuous analogy of moving causes, the first mover argument for the existence of God. Aristotle's argument, as stated in **Physics** VIII and summarized in **Metaphysics** Lambda, **Movement of Animals**, and elsewhere, is a very complex, but the heart of the argument is a continuous analogy, an extension and generalization of the arm — hand — stick example:

> If everything that is in motion must be moved by something, and the movement must either itself be moved by something else or not, and in the former case there must be some first movement that is not itself moved by anything else, while in the case of the immediate movement being of this kind there is no need of an intermediate movement that is also moved (for it is impossible that there should be an infinite series of movements, each of which is itself moved by something else, since in an infinite series there is no first term)—if then everything that is in motion is moved by something, and the first movement is moved but not by anything else, it must be moved by itself.[27]

The first three ways in which Thomas Aquinas proves the existence of God proceed similarly. Particularly close to the Aristotelian argument (it may serve to explicate the passage just quoted) is this argument:

> Whatever is moved must be moved by another. If that by which it is moved be itself moved, then this also must needs be moved by another, and thet by another again. But this cannot go on to infinity, because then there would be no first mover, and consequently, no other mover, seeing that subsequent movers move only inasmuch as they are moved by the first mover... .[28]

Although the argument masquerades as a **taxis**, the regress of moving causes is actually open-ended, because we cannot trace empirically the causes of agents one by one back to God or to the 'first heaven'; without a more adequate cosmology or cosmogony than exists even today it is hazardous to trace moving causes farther than the sun.[29] Supposing one does trace the moving causes to the sun, what the analogy amounts to is the statement that there must be a cause of the sun, and that ultimately the cause of the sun is God, who has no cause. This part of the analogy is purely metaphysical.[30]

If a continuous analogy is to be used as a metaphysically demonstrative argument, either it demonstrates the existence of an infinite series, or there is some member of the series which is necessarily last. Aristotle tries to show that the last terms of his analogies of matter, movement, form and end are indeed necessarily last; his solution is the positing of two metaphysical backstops, in polar opposition: first matter and first mover, and the first mover is also the ultimate final and formal cause.[31]

The polar opposition of ultimate matter and God is developed in terms of his opposition of potentiality and actuality, **dynamis** and **energeia**. The ultimate matter is purely potential, and not actual in any way; God is purely actual, and not potential in any way. This might look like a solution which makes the analogy a **taxis**, but there is a serious difficulty with it: the paradigm cases of 'potentialities' and 'actualities' are all in the observable or 'middle' world, and in this world something has a potentiality **in virtue of** its actuality or **energeia**. Conversely, an **energeia** is precisely the actualization, the activity, the realization of some potentiality which at least coexists the actuality.[32] Thus the existence of something which is pure **dynamis** or pure **energeia** seems ruled out from the start. The concepts of pure potentiality and pure actuality are theoretical, in the sense that their proponent cannot seriously intend them to be independently existent, any more than a Newtonian could seriously assert the independent existence of either force or mass, or an Einsteinian could seriously assert the existence of mass without energy, or energy without mass.

Aristotle would not, I suppose, be overly upset about the description of ultimate matter as a theoretical entity, if we may judge from **Metaphysics** Theta and **Physics** III. He would, however, be upset about the implications for the First Mover, who is otherwise known as God or Mind. If one carries out the program as it begins with biological paradigms, and works up into cosmological applications, the ultimate actuality ought to be the form of the world, the essence of what is; that would be exactly the activity of everything that can become, as it comes into being and passes away. That might be the end point of a **taxis**, an arrangement, of the world. But Aristotle supposes that where there is a **taxis** (or platoon), there must be a **strategos**, (or commander). Thus he pushes his analogy an additional step, from the actuality which is the actualization of **every** potentiality, to the actuality which is the actualization of **no** potentiality. Such an entity could not be a member of that series at all; the continuous analogy is meant to demonstrate the existence of God, but immediately before one arrives at God the analogy becomes **dis**continuous. Plato as metaphysician has a good many faults, but one can say this for him, he never attempts to play this particular trick on us.[33]

II. Interlude: some benign uses of the continuous analogy.

There are two general varieties of relatively benign, as distinguished from malignant, uses of the continuous analogy: one is the use of continuous analogies or proportions in the mathematical analysis and explanation of the world, and the other is the empirical application of continuous analogy in the extended sense.

A. Benign non-mathematical uses of the continuous analogy.

In the non-mathematical, or extended, sense of the word analogy, there are didactic, mnemonic, explanatory, and heuristic uses. Genealogies have been mentioned as examples of the continuous analogy; they may serve as examples of the mnemonic and didactic utilization.

Aristotle's use of analogy in his account of the generation of bees, and in general his sequences of 'causes', are explanatory in intent. Modern biologists too use continuous analogies in explanation:[34] certain structures within single cells are understood as functioning in the cell in a manner comparable to the way cells function in the structures of which they are a part; this continuous analogy may be related to that which argues that certain structures (tissues, organs) function in the body in the way in which certain individuals function in their ecological setting or social group.[35]

Both discrete and continuous analogies are used heuristically, and in the case of the continuous analogy, this is an important use of the open-ended form. The discrete analogy, 'as the heart is to animals with red blood, so X is to animals which do not have red blood,' suggests to Aristotle a proper area of biological investigation. The analogy may lead to some incorrect, but interesting, hypotheses; for example, that all animals have an organ which performs functions analogous to those performed by the heart, or that every animal which has an analogue of the heart has just one. In general, however, such analogies have proven a useful guide to investigation, if occasionally proved fallible.

We may add an example of the heuristic use of the continuous analogy: if the visible objects in the world are composed of molecules, and the molecules are composed of atoms, what are the atoms composed of? One hundred years ago this would have been regarded as a remarkably obtuse question, for few doubted that atoms were the smallest existing material constituents. Today, however, physicists discover particles or quanta which compose the particles which compose the atoms. By making a **taxis** open-ended, a fruitful area of investigation was made available. It might be added that in so doing, investigators were following the suggestion of Plato, in a sense, for that philosopher had said of those things regarded as elementary (**stoicheia**) in his day that they weren't even 'molecules' (first compounds or 'syllables'), **Tim.** 48b.

(II) B. Continuous proportion in the mathematical analysis of the world.

(This section only alludes to some of the investigations of continuous proportion as a principle of mathematical explanation.) Plato's **Timaeus** is, of course, an essay toward the mathematicization of our account of the physical world; continuous proportion plays a vital role in this process. This would I believe, be found true of any of the Pythagoreanizing physicists; one might suggest that the scientists of the Renaissance would be found to make considerable use of continuous proportions. But perhaps this would be expected of a science as highly mathematical, and as conducive to mysticism, as astronomy, the leading science of the Renaissance.

The biological realm too, as D'Arcy Thompson shows in **Growth and Form**, is susceptible to analysis in terms of continuous proportions. One spectacular example is his analysis of 'phyllotaxis' or leaf-arrangement, in which the Fibonacci series (which is determined, as we have noted, by the continuous analogy a : b :: b : a b) plays the central role.[36] If one examines a pine cone, for example, one sees a number of spirals, one set going in one direction around the cone, the other set going in the other direction. One can count these spirals, and find that the numbers of the spirals are three and five, or five and eight. In a daisy, the florets also form spirals, but more, perhaps 13 in one direction, and 21 in the other. In a sunflower the spirals may attain 34 and 55, or 55 and 89. In fact, and in summary (for the evidence I enjoin you to look into Thompson's book), the right- and left-handed spirals in inflorescences are nearly always (over 95 of the time) in ratios composed of numbers in the Fibonacci series. Thompson argues that there is no particular adaptive significance in this series, rather than in some one of the other series which approximate irrational numbers;[37] it might well be a virulent case of mysticism which leads writers like Ghyka to suppose that this arrangement is the major source of the beauty, in our eyes, of such inflorescences.

One may say, more generally, that there is a form of spiral which is closely approximated in a great many phenomena, namely the equiangular (or logarithmic) spiral. This spiral was recognized, and analyzed, by Descartes. If a growing curve cuts

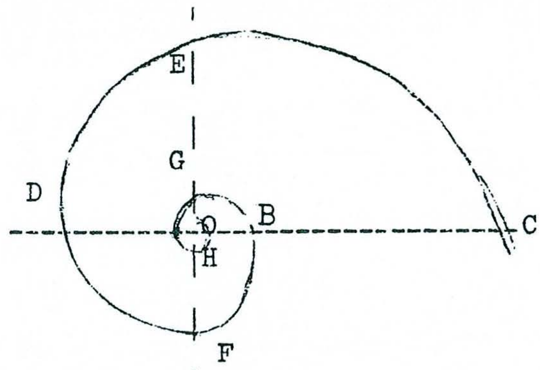


FIGURE 3

> each radius vector at a constant angle— as a circle does... it would necessarily follow that radii at equal angles to one another at the pole would be in continued proportion; and furthermore, that distances measured along the curve from its origin, and intercepted by any radii, as at B, C, are proportional to the lengths of these raddi, OB, OC. It follows that the sectors cut off by successive radii, at equal vectorial angles, are similar to one another in every respect; and it further follows that the figure may be conceived as growing continuously without ever changing its shape the while.[38]

This curve, the equiangular spiral, is just that curve which shells of shellfish describe as they are augmented. This means that the linear characteristics of shellfish along the line of growth are constant, that the plane of the cross-section is in a continuous proportion, and that the volume is in continuous proportion. Thus, although the shell itself is a hard non-living material, as it is augmented it expresses, and preserves in a static form, the proportionality of living growth.

We may, in fact, take this equiangular spiral, as it is exemplified in the conch, as a kind of paradigm of the benign, as distinguished from the malignant, use of the continuous analogy. The tiny embryonic conch, when he began to construct his shell, was very small but had some finite magnitude; he grew according to a continuous proportion, generating his shell according to a complex of equiangular spirals; he died, perhaps prey to a whelk, at some finite size, than which he may have grown larger, had he continued to live. The starting point for the proportion is the size of the shellfish when he starts to build his shell; this size is given rather exactly by the genetic characteristics of his parents. The end point is determined by the death of the conch, which may come at any time, within a limiting span, but must come eventually. There are no infinitesimally small conches, and there are no infinitely large conches, even though the mathematics of their growth, as preserved in their shells, suggests that such creatures might exist, as a theoretical possibility.

III. Critique of the metaphysical use of the continuous analogy.

Neither the discrete nor the continuous analogy causes any very difficult problems when used for arranging what one knows well enough on other evidence, or when used for developing hypotheses for further investigation. The use of equiangular spirals, since Descartes, for the analysis of living structures has an elegance which Plato would have welcomed, and Aristotle would have appreciated. But when an analogy or proportion is used to demonstrate the existence of something **a priori**, when it is used 'metaphysically' in this sense, without any very decisive supporting evidence, problems are bound to arise, and do.

The discrete analogy is not as troublesome as the continuous analogy; in order to construct a discrete analogy, one must have available one ordered pair of terms, and one number of an additional pair, so it is, accordingly, relatively difficult to get very far into the unknown. Indeed, it takes an unusually fertile imagination to get farther than **one** step into the unknown. We might also say that many of the continuous analogies which are clearly organized into **taxeis**, as closed ordered sets, are relatively free of problems, for the **taxis**, at least in its paradeigmatic cases, seems to limit itself. But when an open-ended continuous analogy or proportion is used as a demonstrative argument, one needs only one ordered pair, and their principle of relation, to generate (with the help of a fertile imagination) a whole series of beings, causes, or the like.

(III) A. The dilemma of the continuous analogy.

The philosopher who uses the continuous analogy as a metaphysically demonstrative argument faces a dilemma: either the argument proceeds to demonstrate an infinite series, or it stops in some necessarily last term. In the one case, one must distinguish the argument from vicious regress (or progress), unless one is in the habit of embracing vicious regresses. In the other case, one must show just why the last term is necessarily last.

Mathematical philosophers (strict Aristotelians excepted) have tended to claim that the mathematical use of continuous proportion proceeds to demonstrate **ad infinitum**; this is the principle of mathematical induction.[39] There is nothing objectionable in this procedure so long as the demonstration is not applied to the physical world. But some physicists have argued, from time to time, that matter (or energy) is infinitely divisible on the grounds of a continuous analogy, and the older sort of physics used to claim that there was infinite regress or progress of causes. Leibniz is not alone in supporting that continuous analogies, at least those of certain kinds, **do not** stop. If we take the case of the divisibility of matter as an example, we might distinguish between vicious and non-vicious regress in the following manner: suppose a physicist argues that the characteristics of complexes are determined by the characteristics of simples, and furthermore that matter is infinitely divisible, this would be a vicious regress, because that physicist would be committed to the position that nothing is explicable, because there would be no simples, as grounds of the complex. But the physicist who takes some sort of complex as primary need not about the theoretical possibility of divisibility; he need only claim that the characteristics of the parts depend upon the character and existence of the wholes of which they are, in principle and essentially, parts. In general, a regress is vicious if the ground of understanding or ground of being of the given is thought to be at the end of the regress, and the end of the regress is determined to be at infinity. This is why Plato was so upset about the so-called 'third man argument' in the first part of the **Parmenides**: the forms are supposed to be the ground of intelligibility and ground of being of the world, but if the argument gets started then that ground of intelligibility is at infinity, and thus essentially unintelligible.[40]

The possibility of infinite regresses or processes in the physical, as distinguished from the mathematical, realm is suggested as a consequence of the method of measuring space, or time, or matter,[41] not only in contemporary physics, but also in antiquity. Two of Zeno's paradoxes, the race course, and Achilles and the tortoise, depend upon a principle of measuring, that of 'dichotomy': if there is a finite length, then there is a length which is half (or any nameable fraction) of that length.[42] By this route one arrives at actual infinitesimals; the reverse procedure results in an actual infinite: any finite length can, in principle, be reduplicated **ad infinitum.** These arguments are in the form of continuous analogies: a) $1 : \frac{1}{2} :: \frac{1}{2} : \frac{1}{4} :: \frac{1}{4} : \frac{1}{8} \ldots$; b) $1 : 2 :: 2 : 4 :: 4 : 8 \ldots$. Aristotle does have answers to the problems of dichotomy and of its obverse:

> In the course of a continuous motion the traveller has traversed an infinite number of units in an accidental sense but not in an unqualified sense; for though it is an accidental characteristic of the distance to be an infinite number of half-distances, this is not its real and essential nature.[43]

This answer is clarified in his discussion of infinity,[44] where he concludes that magnitudes are infinite, or infinitesimal, potentially, but not actually (207b12). This, I submit, is the way to deal with all metaphysical uses of the continuous analogy.

As we have seen, Aristotle does not deal skeptically with his continuous analogies of matter, movement, form and end; surely not with those of movement, form and end. Because he wished to assert existences beyond the observable on the basis of continuous analogies, he was caught in the dilemma, and we may see him squirm. He knew all too well that if he allowed a regress of movement, or of form, or of end, it would turn out to be a vicious regress, because the series of movements, and forms, and ends, are meant to be explicative. But as soon as he sought the cause (in the sense of movement, form, or end) of the observable in the unobservable, and sought it with the aid of the continuous analogy, he could not stop, unless he posit God, than whom nothing might be more moving, more formal, or better.

If the use of the concept of analogy by Thomas Aquinas has any validity, it should be in terms of the continuous analogy, rather than in terms of the discrete analogy, although modern Thomists seem not to know this. However, even if Thomas used continuous, rather than discrete, analogies, his enterprise was doomed, just as Aristotle's was doomed, by the horns of the dilemma. At best, the analogy suggests the hypothesis that God exists, it cannot prove it.

The dilemma of the continuous analogy is posed to any philosopher who is to use this form of argument as metaphysically demonstrative. Such a philosopher must choose either to allow the argument to run to infinity, and thus risk either unintelligibility or Ockham's razor or both, or to posit, on more or less flimsy grounds, some member who 'must' be last in the series. Both horns of the dilemma are sharp, both have impaled many a metaphysician already. But the dilemma may be avoided by pure laziness—if you don't exercise the continuous analogy in metaphysical argumentation, it won't hurt you. It is a useful heuristic device, it has explanatory force, but the conch was once a larva, and will someday die.

(III) B. Pure ridicule.

Children of many countries play with sets of dolls, boxes, barrels, or the like, which nest inside each other in a continuous series. If you present such a toy to a child he will remove one doll (or whatever) after the other until he gets to the smallest one, whereupon, if he is a moderately curious child, he will try to extract a still smaller doll. He may not be readily convinced that this is the smallest doll there is; but supposing that he is satisfied on this count, he may put them back together, and when all are assembled, he may ask, 'where is the bigger doll?' 'That's all there were,' you reply. 'There **has** to be a bigger doll,' he says, starting to cry.

When a child looks for a next smaller or next larger member of the series, he is acting reasonably; he has been led to expect such an individual from the character of the other members of the series, and from the character of the relations between them. But the child would be unreasonable, however precocious, if he were to assert that there must be an infinite series of dolls, each one bigger (or smaller) than the next; we would find it equally unreasonable for him to claim that there must be a biggest (or smallest) doll, than which none **could** be bigger (or smaller).

The existence of several members of an ordered series does not demonstrate the existence of further members, but it does suggest the possibility of such members. One cannot conclude from the existence of several members of a continuous series that the series extends to infinity, nor can one, in general, conclude that some member of the series is **necessarily** last, at least not on the basis of the principle of ordering of the series.

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NOTES

1. E.L. Mascall, **Existence and Analogy**, London, 1949; M.-D. Philippe, "**Analogon** and **Analogia** in the Philosophy of Aristotle," **The Thomist** XXXIII (1969) 1-74, for example.
2. For example: Mary Hesse, **Models and Analogies in Science**, Notre Dame, 1966, 130-166; Ernest Nagel, **The Structure of Science**, New York, 1961, 107-117; R. Harre, **Introduction to the Logic of the Sciences**, chapter 4; **et al.**
3. For example, G.E.R. Lloyd, **Polarity and Analogy**, Cambridge, 1966.
4. Philippe, **op. cit.**, mentions only 'arithmetical' analogy in addition to the discrete, pp. 24-28. Lloyd, **op. cit.**, pp. 230, 258, 288, narrowly misses examples of the continuous analogy; see also his "The Role of Medical and Biological Analogies in Aristotle's Ethics," **Phronesis** 13 (1968) 68-83. Also recent, and missing the continuous analogy, Hesse, **op. cit.**, and "Aristotle's Logic of Analogy," **Philosophical Quarterly** 15 (1965) 328-40; "On Defining Analogy," **Proceedings of the Aristotelian Society** 60 (1959—60) 79-100. Some other discussions, also concentrating on the discrete analogy, are: Otto Regenbogen, "Eine Forschungsmethode antiker Naturwissenschaft," **Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik** 1931, 131-182; G.L. Muskens, **De Voce Analogies Significatione ac Usu apud Aristotelem**, Groningen, 1943; Paul Grenet, **Les Origines de l'Analogie Philosophique dans les Dialogues de Platon**, Paris, 1948; Jan Van der Meulen, **Aristotles, die Mitte in Seinem Denken**, Meisenheim, 1951, p. 4, 144, 185ff.
5. D'Arcy W. Thompson, "Excess and Defect," **Mind** 38 (1928), reprinted in **Science and the Classics**, OUP 1940, pp. 188-213. F.M. Cornford, **Plato's Cosmology**, Cambridge 1937, pp. 43-52. Dorothy Emmet, in "The use of Analogy in Metaphysics," **Proceedings of the Aristotelian Society** 41 (1940—41) 27-46, discusses Plato's line passage in **Republic** VI as a discrete analogy; on p. 28 she gives an example of a continuous analogy, without distinguishing it from a discrete analogy. R.S. Brumbaugh, **Plato's Mathematical Imagination**, Bloomington, Indiana, 1954, especially chapter IV. In the background of my interpretation of Plato's use of continuous analogy are the works of Matila Ghyka, especially **Le Nombre d'Or**, Paris 1931; **The Geometry of Art and Life**, which is an

English summary of **Esthetique des Proportions dans la Nature et dans les Arts,** Paris, 1927; I have had use of the Spanish translation of this last work, **Estetica de las Proporciones en la Naturaleza y en las Artes,** Buenos Aires, 1953. I hereby thank Mr. Lawrence Ault, a student in one of my courses, for making me acquainted with Ghyka's works. Some writers have recognized the existence of continuous analogies in presocratic philosophers: H. Frankel, "A thought pattern in Heraclitus," **American Journal of Philosophy** 59 (1935) 309-337; Charles Kahn, **Anaximander and the Origins of Greek Cosmology,** New York, 1960, pp. 170ff.

6. See Martin Ostwald, **Aristotle: Nicomachean Ethics,** N.Y., 1962, p. 120, n. 24; R.A. Gauthier et J.Y. Jolif, **L'Ethique a Nicomaque,** Tome II, Louvain and Paris, 1959, pp. 138, 357; H.H. Joachim, **Aristotle: The Nicomachean Ethics,** Oxford, 1951, 142-3, 147; W.D. Ross, **Ethica Nicomachea,** Oxford, 1925; W.F.R. Hardie, "Aristotle's Doctrine that Virtue is a Mean," **Proceedings of the Aristotelian Society** 65 (1964—5) 183-204, especially p. 200.
7. It is also used at II.5, 1106a29-36.
8. See Brumbaugh, **op. cit.,** 209-229; Cornford, **op. cit.,** 66-72. Later Neopythagoreans and Neoplatonists used some 10 different forms of **analogia** or proportion. See Ghyka, **Nombre d'Or,** p. 32.
9. Cornford translation. Brumbaugh comments upon this passage thus, p. 214:

   Two classes may be 'bound' or 'connected' by the creation of an intermediate class between them, similar to both. Like entities fuse together, unlike separate. Thus if the two unlike and extreme classes, **a** and **c**, are brought into contact, they will not hold together. If, on the other hand, **a** is brought into contact with a like class **b**, and **b** with **c**, which it is like, the chain **a-b-c** will hold. Terms in proportionate relation cannot be disconnected by reversal, translation, or any external force causing a transposition, since any such force will have the same effect on all similar entities on which it acts, and will therefore exert the same transposing effect on both sides of the proportion.

   Perhaps this will clarify for the reader Plato's text, although I find Plato more perspicuous.

   See also Charles Mugler, **La Physique de Platon,** Paris, 1960, who discusses this passage as a **taxis** and as a mean proportion, p. 8, p. 42, and elsewhere.
10. Empedocles, frag. 236 (DK 31 B 82).
11. Plato, **Republic** IV, 444c.
12. Aristotle, **PA** I.5, 645b17. Lloyd is a good source for further examples of discrete analogies, especially those used by Aristotle, but also some used by other ancient authors.
13. I draw the inference from **GA** III.10, 762a32, that Aristotle means to use this word in a technical way for analogies of the sort which we here describe. The first set of senses of the word **taxis** listed in LSJ are in reference to military orderings of one sort or another; the presocratic philosophers (DK register p. 422) and Plato (e.g. at

Tim. 30a) use the word of the 'ordering' of the cosmos, as does Aristotle in **Met.** Lambda 10, 1075a12ff. The **Timaeus** use of the word gets some of its force, I think, from the argument in the **Gorgias** from about 503e to 506d; Socrates there argues that any **demiourgos** contributes to his work by disposing elements in a fixed **taxis,** and "compels one to fit and harmonize with the other until he has combined the whole into something well ordered and harmonized." Thus "harmony and **taxis**" make something useful and beautiful, and this is supposed to hold as well for the soul. In **Republic** IX, 577d2, Plato argues that "if man resembles the state, the same **taxis** must obtain in him... ." In this place Paul Shorey translates the word **'taxis'** as 'proportion', and this would fit the theory of the parts of the soul, and the parts of the state, which I understand Plato to be arguing in this work. See also **Laws** 2, 653e4; 10, 898b1; **Epinomis** 982a7.

14. **GA** III.10, 759a8ff.
15. The 'argument' is called an **análogón pos** at 760a13, and a **taxis** at 762a32.
16. A.L. Peck, **Aristotle: Generation of Animals,** London, 1953.
17. **GA** I.22, 730b18. Patterson Brown, "Infinite Causal Regression," **Philosophical Review** 75 (1966) 519-525, shows how series of moving causes are made into what we call open-ended continuous analogies; he does not comment on the analogical character of such arguments however.
18. **Met.** Lambda 10, 1075a12ff, my translation, with thanks to those of Ross and Tredennick.
19. **Phaedo** 62b.
20. The ramifications of this analogy are explored by E.N. Lee, "On the Metaphysics of the Image in Plato's **Timaeus,**" **The Monist** 50 (1966) 341-368. See also his "On Plato's **Timaeus,** 49d4-e7," **American Journal of Philosophy** 88 (1967) 1-28.
21. See Brumbaugh, **op. cit.**, pp. 38-44; Ghyka, **Nombre d'Or** ch. I, and **Estetica de las Proporciones.** The method of generating irrational numbers as diagnoals of squares to which Plato alludes in this place in the **Theaetetus,** and which Euclid too ascribes to Theaetetus, generates the Golden Number at least as easily and quickly as it generates 17.
22. Some explanation of the word **'gnomon'** in this context might be helpful. A **gnomon** is a geometrical figure (of two of three dimensions, usually) which, when added to an original geometrical figure, generates another figure similar to the original figure. In Pythagorean mathematics, the **gnomon** of an equiangular triangle is a line one unit longer than the side of the original triangle; the **gnomon** of a square is a right angle whose legs are one unit longer than the sides of the original square. See Aristotle, **Categories** 15a30; **Physics** III.4, 203a14. But it is possible to have a gnomon which is not a line but an area. Every triangle has another triangle which is its gnomon, which when added to it, makes another similar triangle. (See Ghyka, **Estetica, p.** 127). Constructing the successive gnomons of the triangle generates an approximation of

a spiral.

It is peculiar to the **phi** rectangle to have the square as its gnomon; Ghyka makes much of this in the analysis of the aesthetics of architectural compositions. Thompson too, in **Growth and Form** pp. 759-766, uses **phi** progressions for the generation of spirals.

Incidentally, the rectangle of 1 X 2 is its own gnomon, or 1 : 2 :: 2 : 2.

23. Cf. I. **Chron.** or **Matt.** I.1-16.
24. See, for example, **GA** II.1, 731b32ff: since the nature of this sort of class cannot be eternal, that which is generated is eternal in the way which is possible. It is impossible numerically, for the entity of the beings is in the individuals; if it were thus, they would be eternal; **but it is possible for a species.** Therefore there is always a race of men and animals and plants... . (after Peck)
25. **Met.** Theta 7, 1049a19ff, Ross translation. There is another continuous analogy of materials, done from a biological point of view, in **PA** II.1, 646a13-24; in this case the stopping point is the elementary powers (hot, cold, wet, dry) rather than 'first matter', For more on the notion of prime matter in Aristotle, see my "Science and Philosophy in Aristotle's **Generation of Animals,**" **Journal of the History of Biology,** Spring 1970.

    Aristotle may recognize the open-endedness of this analogy at **Physics** III.7, 207b35, where he says, "in the four-fold scheme of causes, it is plain that the infinite is a cause in the sense of matter... ." It wouldn't do his theory much harm if he were to let the analogy of materials run, since he doesn't suppose that reduction to material elements is a primary mode of understanding. It is the complex of analogies which lead toward the prime mover that must give him most worry, for he must find a source of movement, and a form, and a purpose, for the universe, if he can make good his claim against Democritus that the world is not here 'by chance'.
26. Cf. **MA** 10, 703a30.
27. **Phys.** VIII.5, 256a13-21, Hardie and Gaye translation. For an analysis of the argument, taking cognizance of its complexity, see I.P. McGreal, Analyzing **Philosophical Arguments,** San Francisco, 1967, pp. 297-333. McGreal does not, however, note the analogical character of the argument.
28. **Summa Theologica** Question 2, Article 3. Plato **Laws** X. 894 ff also has a version of the first mover argument.
29. Plato **Rep.** VI, 509b; Ar. **Met.** Lambda 5, 1071a15, **GA** I.2, 716a16, **GC** II.11, 338b3; these are all examples of stopping a series of moving causes with the sun.
30. The argument from design, although usually stated as a discrete analogy, may be stated as a continuous analogy— artifacts : man :: man : God (this is a **taxis** argument). In the discrete form it is attacked quite nicely by David Hume in **Dialogues Concerning Natural Religion.** His criticisms would, I think, apply as well to the

continuous form.

31. Patterson Brown, **op. cit.**, shows how this works out, in a general way, and it is both obvious and well-known.

    Evert W. Beth, **The Foundations of Mathematics**, N.Y. 1964, pp. 9-12, calls attention to "The Principle of the Absolute," which is the principle that continuous analogies have to end **somewhere**, so justifying the positing of something, The Absolute, as the end of such analogies. Beth identifies this principle with Aristotle's argument for the prime mover, and argues against it as a mathematical principle; his argument is not dissimilar from that which is suggested here. But Beth does not use a similar argument against the reciprocal principle, which might be known as "The Principle of the Infinite," probably because mathematicians have a good deal of use for infinities, and very little use for absolutes.
32. Gold can be made into coins, is potentially coins, in virtue of the already existent degree of hardness and malleability of gold, and this hardness and malleability is already an actuality. The coin as actuality is the realization of the potentialities still present in it as a **gold** coin. But this sketch is the least possible which could be offered. For a further discussion of these terms, see "Science and Philosophy in Aristotle's **Generation of Animals**," especially part C. Aristotle's analysis of **dynamis** and **energeia** is highly complex; the present account aims at less, rather than more, unfairness to him.
33. Plato's argument in **Laws** X is meant to demonstrate the existence of a soul of the world, an **anima mundi**, which is rather similar to the notion of an actualization of all potentialities, in the way in which Aristotle's 'soul' in **De Anima** II is the first actualization of a natural body having the potentiality for life.

    Incidentally, Aristotle has a temporizing measure in his continuous analogy of movements, a device which may be called the 'cyclical' analogy. Its form would be a : b :: b : c :: c ... :a. There is no regress here, but some might suppose the circle vicious. In fact, Aristotle puts any malignancy on a higher level, for he explains the actuality of such circles by reference to a first mover, so we return to the metaphysical backstop.
34. E.g., J.T. Bonner, **The Ideas of Biology**, N.Y. 1962, 109ff.
35. Cf. Aristotle **EN** I.7, 1097b3; **Pol.** III.4, 1277a5ff; Lloyd, "The role of medical and biological analogies in Aristotle's ethics," **Phronesis** 13 (1968) 68-83.
36. D.W. Thompson, **Growth and Form** 2nd ed., Cambridge, 1952; ch. XIV, pp. 912-933. He refers to the earlier, and extensive, literature on the subject.
37. These series are laid out nicely in his "Excess and Defect."
38. Thompson, **Growth and Form**, p. 754. I also follow him for the ascription of the analysis of the equiangular spiral to Descartes. He refers to the Adam and Tannery edition of the Oeuvres, Paris, 1898, p. 360; the edition of Descartes which I possess does not seem to have the letter in question, one to Mersenne in 1638. The illustration in the text, figure 3, is Thompson's.

39. Cf. Bertrand Russell, **Introduction to Mathematical Philosophy**, Ch. III; W. Kneale, **Probability and Induction**, pp. 37-43.
40. **Parmenides** 132; cf. **Republic** X.597, **Timaeus** 31a. There is, of course, an indefinitely large literature on this topic, reviewed most recently by Gregory Vlastos, in "Plato's 'Third Man' argument (**Parm.** 132al-b2): text and logic," **The Philosophical Quarterly** 19 (1969) 289-301.
41. Cf. Henry Morgenau, **The Nature of Physical Reality**, N.Y. 1950, pp. 163-4.
42. Aristotle **Physics** VI.9, 239b5ff; Simplicius **Physics**, 1013,31; 1289,5. See also Gregory Vlastos, "Zeno's Race Course," **Journal of the History of Philosophy** 1966, 95-108.
43. **Physics** VIII.8, 263b6-9, Hardie and Gaye translation.
44. **Physics** III.4-8, 202b30-208a25; Cf. **Metaphysics** Kappa 10, 1066a35-1068b7.

I take this opportunity to thank all those who have read earlier versions of this paper for their comments and suggestions, and to thank the Creighton Club for permission to publish here those sections of this paper which have appeared in its **Proceedings** for 1969—70.